**Master equation**

In biochemical systems where the copy number of each molecular specimen can be low, it might be necessary to use a stochastic description of the system. The master equation is a time-dependent equation for the probability density \( p(x,t) \) of the chemical species in the system. It gives a description of the system on a mesoscopic level.

Master equation for a model with \( R \) reactions:

\[
\frac{\partial p(x,t)}{\partial t} = \sum_{i=1}^{R} \left( \frac{\partial}{\partial x} \left( \alpha_i(x) \eta_i \right) \right) p(x,t) - \sum_{i=1}^{R} \frac{\partial}{\partial x} \left( \alpha_i(x) \right) p(x,t)
\]

The vector \( x \) is the state of the system, defined by the amounts of the chemical species, causing the dimensionality of the equation to increase with the number of reactants. The parameter \( \alpha \) is the probability of a state change through reaction \( i \).

**Linear noise approximation**

- The linear noise approximation (LNA) is an expansion of the master equation in an expansion parameter, \( \Omega \), corresponding to the volume of the system.

\[
\bar{x} = \Sigma \phi_i + \Omega^\frac{1}{2} \Sigma \phi_i^{\prime}
\]

- The approximation results in each state being described through two variables; the mean number variable, \( \bar{x} \), and the fluctuation variable, \( \delta_x \).

- Rewriting the master equation in the new variables, one can separate the equation into two parts, based on different orders of \( \Omega \). One part describes changes in the mean number variable and the other shows the probability distribution of the fluctuation variable over time.

**Separation of timescales**

To improve the LNA a separation of timescales was performed. This method is based on a change into variables that differ considerably in their rates of reaction. The method requires that the slow variable in the new system is a linear combination of the original variables, and the fast variable is at its steady-state value, conditional on the slow variable. One can thereafter study the two variables separately.

**Results**

- The systems show the same dynamic behavior with the LNA as with the Fokker-Planck approximation. The fluctuation variables were plotted as in figure 2, showing the fluctuations from the mean value over time.

- Performing LNA on a multidimensional system, with only a few variables studied with fluctuations, largely reduces the computational demands compared to using the Fokker-Planck approximation on the full problem.

- Using the separation of timescales overcomes the drawback of having a linear distribution from the LNA and its known non-linear system.