Resolving degeneracies with future LBL experiments on different scales of $\sin^2 2\theta_{13}$

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- Introduction
- Impact factors to the measurement
- General properties of the appearance channels
- Different scales of $\sin^2 2\theta_{13}$ and experiments to access them
- Resolving degeneracies by the combination of experiments on different scales of $\sin^2 2\theta_{13}$
- Summary and conclusions
Introduction: neutrino oscillations

\[ P_{\alpha\beta} = \delta_{\alpha\beta} - 4 \sum_{i=1}^{3} \sum_{j=1}^{3} \text{Re} J_{ij}^{\alpha\beta} \sin^2 \Delta_{ij} - 2 \sum_{i=1}^{3} \sum_{j=1}^{3} \text{Im} J_{ij}^{\alpha\beta} \sin 2\Delta_{ij} \]

with

\[ J_{ij}^{\alpha\beta} \equiv U_{\alpha i} U_{\alpha j}^{*} U_{\beta i} U_{\beta j} \]

\[ \Delta_{ij} \equiv \frac{\Delta m_{ij}^2 L}{4E} \equiv \frac{(m_i^2 - m_j^2) L}{4E} \]

\[ U = \begin{pmatrix}
    c_{12} c_{13} & s_{12} c_{23} - c_{12} s_{23} s_{13} e^{i\delta} & s_{12} c_{23} - s_{12} s_{23} s_{13} e^{i\delta} \\
    -s_{12} c_{23} - c_{12} s_{23} s_{13} e^{i\delta} & c_{12} c_{23} & s_{23} c_{13} \\
    s_{12} s_{23} - c_{12} c_{23} s_{13} e^{i\delta} & -c_{12} s_{23} - s_{12} c_{23} s_{13} e^{i\delta} & c_{23} c_{13}
\end{pmatrix}. \]

\[ \rightarrow \text{Six parameters: } \Delta m_{31}^2, \Delta m_{21}^2, \theta_{23}, \theta_{12}^2, \theta_{13}, \delta_{\text{CP}} \]
Classification of oscillation parameters

... with mass hierarchy $|\Delta m^2_{21}| \ll |\Delta m^2_{31}|$ and $\sin^2 2\theta_{13} \lesssim 0.1$:

- Atmospheric oscillations
  - $\Delta m^2_{31}$
  - $\theta_{23}$

- Solar oscillations
  - $\Delta m^2_{21}$
  - $\theta_{12}$

Small "reactor" angle $\theta_{13}$

CP effects $\delta_{CP}$

Future important experiments:
- Beams (K2K, MINOS, CNGS)
- Reactor ($\theta_{13}$)
- Superbeams
- $\nu$-Factories
- Beams?
- Solar exp.
- Superbeams
- $\nu$-Factories

Most interesting for future LBL: $\theta_{13}, \text{sgn}(\Delta m^2_{31}), \delta_{CP}$
Common features of future LBL exp.

- L, E chosen that in oscillation maximum:
  \[ \Delta m^2 L/E = \mathcal{O}(1) \]
  \( \rightarrow L \sim 1 \text{ km} - 8000 \text{ km} \)
  \( \rightarrow E \sim 1 \text{ MeV} - 50 \text{ GeV} \)

- Artificial neutrino source:
  Reactor, Accelerator
  \( \rightarrow \) well-known flux

- Often: near detector for better control of systematics
  (for hadronic prod. of \( \nu \)'s)
### Different types of future LBL exp.

<table>
<thead>
<tr>
<th></th>
<th>Reactor exp.</th>
<th>Superbeam</th>
<th>Neutrino Factory</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Timescale</strong></td>
<td>≲ 2010</td>
<td>≲ 2015</td>
<td>&gt; 2015</td>
</tr>
<tr>
<td><strong>Source</strong></td>
<td>Nuclear fission reactor</td>
<td>Accelerator</td>
<td>Accelerator</td>
</tr>
<tr>
<td><strong>ν-production</strong></td>
<td>Hadronic</td>
<td>Hadronic</td>
<td>Leptonic</td>
</tr>
<tr>
<td></td>
<td>→ near detector</td>
<td>→ near detector</td>
<td></td>
</tr>
<tr>
<td><strong>Constraints</strong></td>
<td>Systematics</td>
<td>Systematics</td>
<td>Statistics</td>
</tr>
<tr>
<td><strong>Challenges</strong></td>
<td>Knowledge of detector</td>
<td>Backgrounds</td>
<td>Target power, muon cooling</td>
</tr>
<tr>
<td><strong>ν-energies</strong></td>
<td>≈ 4 MeV</td>
<td>≈ 1 GeV</td>
<td>≈ 50 GeV</td>
</tr>
<tr>
<td><strong>Baselines</strong></td>
<td>1 – 2 km</td>
<td>300 – 1 000 km</td>
<td>700 – 7 500 km</td>
</tr>
</tbody>
</table>

Resolving degeneracies ... – Walter Winter – p.6
Core element: Muon storage ring with straight decay sections
Impact factors to the measurement

1. Statistical errors

\[ \chi^2 \]

\[ \sin^2 2\theta_{13} \]

2. Systematics

\[ \chi^2 \]

\[ \sin^2 2\theta_{13} \]

3. Correlations

\[ \Delta m^2_{13} \]

\[ \sin^2 2\theta_{13} \]

4. Degeneracies

\[ \sin^2 2\theta_{13} \]

\[ \delta_{CP} \]

5. External input

\[ \Delta m^2_{13} \]

\[ \sin^2 2\theta_{13} \]

6. True values

\[ \text{True value of } \sin^2 2\theta_{13} \]

\[ \text{True value of } \Delta m^2_{13} \]

Determined by R&D of experiment

Controllable by L, E, combinations, ...

No influence by experiment

For a more detailed discussion: see Secs. 3 and 5 of hep-ph/0204352
Example: $\sin^2 2\theta_{13}$-sensitivity limit

... for individual experiments:

The appearance channels

LBL exp: interesting information in $P_{\text{app}} = P_{e\mu}, P_{\mu e}, P_{e\mu}$, or $P_{\mu e}$

To second order in $\sin 2\theta_{13}$ and the hierarchy parameter $\alpha \equiv \frac{\Delta m_{21}^2}{\Delta m_{31}^2}$:

$$P_{\text{app}} \approx \sin^2 2\theta_{13} \sin^2 \theta_{23} \frac{\sin^2[(1 - \hat{A})\Delta]}{(1 - \hat{A})^2}$$

$$\pm \alpha \sin 2\theta_{13} \xi \sin \delta_{CP} \sin(\Delta) \frac{\sin(\hat{A}\Delta)}{\hat{A}} \frac{\sin[(1 - \hat{A})\Delta]}{(1 - \hat{A})}$$

$$+ \alpha \sin 2\theta_{13} \xi \cos \delta_{CP} \cos(\Delta) \frac{\sin(\hat{A}\Delta)}{\hat{A}} \frac{\sin[(1 - \hat{A})\Delta]}{(1 - \hat{A})}$$

$$+ \alpha^2 \cos^2 \theta_{23} \sin^2 2\theta_{12} \frac{\sin^2(\hat{A}\Delta)}{\hat{A}^2}$$,

$$\Delta \equiv \frac{\Delta m_{31}^2 L}{4E}, \xi \equiv \cos \theta_{13} \sin 2\theta_{12} \sin 2\theta_{23}, \hat{A} \equiv \pm \frac{2\sqrt{2}G_F n_e E}{\Delta m_{31}^2}.$$
Problems with degeneracies

Especially for large $\alpha$ all terms act simultaneously
→ A different parameter value in one term can often be compensated by a different parameter value in another term
→ There exists an “eight-fold” degeneracy (Barger, Marfatia, Whisnant, 2001):

1) $\text{sgn}(\Delta m^2_{31})$-degeneracy (Minakata, Nunokawa, 2001)
   Most important for us: solution for opposite sign of $\Delta m^2_{31}$ spoils especially mass hierarchy and $\sin^2 2\theta_{13}$ measurements

2) $(\theta_{23}, \frac{\pi}{2} - \theta_{23})$-degeneracy (Fogli, Lisi, 1996)
   Does not appear for current best-fit value $\theta_{23} = \pi/2$

3) $(\delta, \theta_{13})$-degeneracy (Burguet-Castell, Gavela, Gomez-Cadenas, Hernandez, Mena, 2001)
   Important for neutrino factories because of good energy resolution and statistics
The parameters of interest (illustrated)

\[ P_{\text{app}} \approx \sin^2 2\theta_{13} \sin^2 \theta_{23} \frac{\sin^2 [(1 - \hat{A})\Delta]}{(1 - \hat{A})^2} \]

- Main information in first term
- First term enhanced by matter effects (Resonance: \( \hat{A} \rightarrow 1 \))
- Spoilt by correlations and degeneracies for large \( \alpha \)

![Diagram showing the presence of degeneracies and the sensitive region](image)

\[ + \alpha \sin 2\theta_{13} \xi \sin \delta_{\text{CP}} \sin(\Delta) \frac{\sin(\hat{A}\Delta)}{\hat{A}} \frac{\sin[(1 - \hat{A})\Delta]}{(1 - \hat{A})} \]

\[ + \alpha \sin 2\theta_{13} \xi \cos \delta_{\text{CP}} \cos(\Delta) \frac{\sin(\hat{A}\Delta)}{\hat{A}} \frac{\sin[(1 - \hat{A})\Delta]}{(1 - \hat{A})} \]

\[ + \alpha^2 \cos^2 \theta_{23} \sin^2 2\theta_{12} \frac{\sin^2 (\hat{A}\Delta)}{\hat{A}^2} \]
The parameters of interest (illustrated)

\[ \text{sgn}(\Delta m_{31}^2) \]

- Main information in first term through matter effects
- Biggest influence at resonance \( \hat{A} \to 1 \)
- Spoilt by \( \text{sgn}(\Delta m_{31}^2) \)-degeneracy for large \( \alpha \)
- True value of \( \delta_{\text{CP}} \) relevant!

\[
P_{\text{app}} \approx \sin^2 2\theta_{13} \sin^2 \theta_{23} \frac{\sin^2[(1 - \hat{A})\Delta]}{(1 - \hat{A})^2}
\]

\[
\pm \alpha \sin 2\theta_{13} \xi \sin \delta_{\text{CP}} \sin(\Delta) \frac{\sin(\hat{A}\Delta)}{\hat{A}} \frac{\sin[(1 - \hat{A})\Delta]}{(1 - \hat{A})}
\]

\[
+ \alpha \sin 2\theta_{13} \xi \cos \delta_{\text{CP}} \cos(\Delta) \frac{\sin(\hat{A}\Delta)}{\hat{A}} \frac{\sin[(1 - \hat{A})\Delta]}{(1 - \hat{A})}
\]

\[
+ \alpha^2 \cos^2 \theta_{23} \sin^2 2\theta_{12} \frac{\sin^2(\hat{A}\Delta)}{\hat{A}^2}
\]
The parameters of interest (illustrated)

$\delta_{CP}$

Main information in second and third terms

Other terms act as background

Large $\sin 2\theta_{13}$ and $\Delta m_{21}^2$ required

$$P_{app} \simeq \sin^2 2\theta_{13} \sin^2 \theta_{23} \frac{\sin^2[(1 - \hat{A})\Delta]}{(1 - \hat{A})^2}$$

$$\pm \alpha \sin 2\theta_{13} \xi \sin \delta_{CP} \frac{\sin(A\Delta)}{A} \frac{\sin[(1 - \hat{A})\Delta]}{(1 - \hat{A})}$$

$$+ \alpha \sin 2\theta_{13} \xi \cos \delta_{CP} \frac{\cos(A\Delta)}{A}$$

$$+ \alpha^2 \cos^2 \theta_{23} \sin^2 2\theta_{12} \frac{\sin^2(A\Delta)}{A^2}.$$
**Different scales of** $\sin^2 2\theta_{13}$

**True value of** $\sin^2 2\theta_{13}$:

<table>
<thead>
<tr>
<th>$10^{-1}$</th>
<th>$10^{-2}$</th>
<th>$10^{-3}$</th>
<th>$10^{-4}$</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>CHOOZ bound</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Timescale???

<table>
<thead>
<tr>
<th>2000</th>
<th>2010</th>
<th>2025</th>
<th>2040</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>CHOOZ bound</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Sensitive Experiments:**

- Conventional
- Superbeams
- $\nu$-Factories
- $\nu$-Factories?
- Reactor
- $\nu$-Factories
- Reactor
- Superbeams

→ Experiments to resolve degs. “selected” by true value of $\sin^2 2\theta_{13}$
→ Combinations of experiments with similar capabilities!
Resolving degs: $\sin^2 2\theta_{13} \sim 10^{-2} - 10^{-1}$

Options:

- **Combinations of first-generation superbeams**
  - (Whisnant, Yang, Young, 2002)
  - (Barger, Marfatia, Whisnant, 2002)
  - (Minakata, Nunokawa, Parke, 2003)

- **Reactor experiments and Superbeams**
  - (Minakata, Sugiyama, Yasuda, Inoue, Suekane, 2002)
  - (Huber, Lindner, Schwetz, Winter, 2003, hep-ph/0303232)

- **Others!?**
## First-generation superbeams

<table>
<thead>
<tr>
<th></th>
<th>JHF-SK</th>
<th>NuMI</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Beam</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Baseline</td>
<td>295 km</td>
<td>712 km</td>
</tr>
<tr>
<td>Target Power</td>
<td>0.77 MW</td>
<td>0.4 MW</td>
</tr>
<tr>
<td>Off-axis angle</td>
<td>2°</td>
<td>0.72°</td>
</tr>
<tr>
<td>Mean energy</td>
<td>0.76 GeV</td>
<td>2.22 GeV</td>
</tr>
<tr>
<td>Mean $L/E$</td>
<td>385 km GeV$^{-1}$</td>
<td>320 km GeV$^{-1}$</td>
</tr>
<tr>
<td><strong>Detector</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Technology</td>
<td>Water Cherenkov</td>
<td>Low-Z calorimeter</td>
</tr>
<tr>
<td>Fiducial mass</td>
<td>22.5 kt</td>
<td>17 kt</td>
</tr>
<tr>
<td>Running period</td>
<td>5 years</td>
<td>5 years</td>
</tr>
</tbody>
</table>

(Itow et al, 2001; Ayres et al, 2002)
JHF-SK and NuMI combined

Initial situation as in LOIs:
- No sensitivity to the mass hierarchy
- Both optimized for similar params
- Only marginal CP sensitivity

Possible modifications:
- For $\delta_{CP}$: partial antineutrino running (especially JHF-SK!)
- For mass hierarchy:
  NuMI@longer baseline
  (fixed decay pipe $\rightarrow$ 890 km
  max. baseline for OA $0.72^\circ$)

(90% CL, $\Delta m^2_{31} = +3 \cdot 10^{-3} \text{ eV}^2$, $\sin^2 2\theta_{23} = 1$, $\sin^2 2\theta_{12} = 0.8$, $\delta_{CP} = \pi/2$ for CP violation, or conservative choice for mass hierarchy)

Reactor experiments

... with near and far detectors as alternative options to measure $\sin^2 2\theta_{13}$

Examples:

<table>
<thead>
<tr>
<th></th>
<th>Reactor-I</th>
<th>Reactor-II</th>
</tr>
</thead>
<tbody>
<tr>
<td>Integrated luminosity</td>
<td>400 t GW y</td>
<td>8000 t GW y</td>
</tr>
<tr>
<td>Unoscillated events</td>
<td>31,493</td>
<td>629,867</td>
</tr>
<tr>
<td>$\sigma_{\text{norm}}$</td>
<td>0.8%</td>
<td>0.8%</td>
</tr>
<tr>
<td>$\sigma_{\text{cal}}$</td>
<td>0.5%</td>
<td>0.5%</td>
</tr>
<tr>
<td>Baseline</td>
<td>1.7 km</td>
<td>1.7 km</td>
</tr>
<tr>
<td>Detector equivalent</td>
<td>$4 \times$ CHOOZ</td>
<td>KamLAND</td>
</tr>
<tr>
<td>(for 2 y and 10 GW)</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

(Luminosities given in detector mass [t] $\times$ thermal reactor power [GW] $\times$ running time [y])

→ “Clean” measurement without correlations/degeneracies!
Reactor experiments versus superbeams

(90% CL, $\Delta m^2_{31} = +3 \times 10^{-3}$ eV$^2$, $\Delta m^2_{21} = 7 \times 10^{-5}$ eV$^2$, $\sin^2 2\theta_{23} = 1$, $\sin^2 2\theta_{12} = 0.8$)

$\rightarrow$ Reactor experiments less dependent on true values of $\Delta m^2$'s

(Huber, Lindner, Schwetz, Winter, 2003, hep-ph/0303232)
Reactor experiments plus superbeams

Reactor-II plus two (optimized) superbeams very competitive

Reactor-II plus neutrino running better than combined neutrino-antineutrino running (JHF-SK_{cc})

(Huber, Lindner, Schwetz, Winter, 2003, hep-ph/0303232)
Resolving degs: \( \sin^2 2\theta_{13} \sim 10^{-3} - 10^{-2} \)

Options:

- **Combinations of superbeam upgrades**
  
  (Barger, Marfatia, Whisnant, 2002)

- **Superbeam upgrade and neutrino factory**
  
  (Burguet-Castell, Gavela, Gomez-Cadenas, Hernandez, Mena, 2002)

- **Superbeam at “magic baseline” (cf., later)**
  
  (Asratyan, Davidenko, Dolgolenko, Kaftanov, Kubantsev, Verebryusov, 2003)

- **Neutrino factory with “silver channels” (\( \nu_\tau \))**
  
  (Donini, Meloni, Migliozzi, 2002)

- **Others!?**
Resolving degs: \( \sin^2 2\theta_{13} \sim 10^{-4} - 10^{-3} \)

<table>
<thead>
<tr>
<th>CHOOZ bound</th>
</tr>
</thead>
</table>

\[
\begin{array}{cccc}
10^{-1} & 10^{-2} & 10^{-3} & 10^{-4} \\
\end{array}
\]

\( \sin(\hat{A}\Delta) \equiv 0 \): “Magic baseline”

\( \hat{A} \rightarrow 1 \): Matter resonance

\rightarrow Correlations/degs. disappear

\rightarrow Independent of \( E \), osc. params

\rightarrow Evaluates to \( \sqrt{2}G_F n_e L = 2\pi \),

\( L_{\text{magic}} \approx 7\,630 \text{ km} \) (average density)

\( L_{\text{magic}} \approx 7\,250 \text{ km} \) (PREM profile)

However: no CP at \( L_{\text{magic}} \)!

Other options with two detectors: silver channels!?, ...
Example: NuFact-II

... as a “large” neutrino factory

- Detector: magnetized iron calorimeter
- Detector mass: 50 kt
- Running time: 4 y neutrinos plus 4 y antineutrinos
- Baseline: 3 000 km
- Target power: 4 MW
  \( \sim 5.3 \cdot 10^{20} \) useful muon decays/year
- 5% matter density uncertainty assumed

Now: split detector mass into two equal pieces of 25 kt placed at \( L_1 \) and \( L_2 \)
Full analysis and the magic baseline

... for $\sin^2 2\theta_{13}$-sensitivity limit in two baseline space ($L_1, L_2$)
(incl. correlations and degeneracies)

Three local minima:
(1) $L_1 \simeq L_2 \simeq 7500$ km:
→ No CP measurement possible!
(2) $L_1 \simeq 7500$ km, $L_2 \simeq 3000$ km:
→ Stable minimum
→ Independent of osc. parameters
(3) $L_1 \simeq 4750$ km, $L_2 \simeq 2250$ km:
→ Preferred by statistics
→ Depends on osc. parameters
→ Unstable ($\delta_{CP}, \theta_{13}$)-degeneracy

Magic baseline: Main results

Magic baseline $L \sim 7\,500\,\text{km}$

compared to $L = 3\,000\,\text{km}$:

a) $L_1 = L_2 = 3\,000\,\text{km}$:
   - Poor $\text{sgn}(\Delta m_{31}^2)$ measurement
   - High risk of unknown $\Delta m_{21}^2$

b) $L_1 = L_2 = 7\,500\,\text{km}$
   - No CP sensitivity

c) $L_1 = 7\,500\,\text{km}$, $L_2 \sim 3\,000\,\text{km}$:
   - Excellent for all parameters

(Figure: $3\sigma$ CL; $\delta_{CP} = +\pi/2$ for CP violation and varied for $\text{sgn}(\Delta m_{31}^2)$; red bars by variation of
$\Delta m_{21}^2$ in KamLAND-allowed $3\sigma$-range; arrows = LMA best-fit $\Delta m_{21}^2 \sim 7 \cdot 10^{-5} \,\text{eV}^2$)

Discussion based upon true value of $\sin^2 2\theta_{13}$
- “Selects” the experiments, which are sensitive
- These can be combined to resolve degeneracies
- Different from a strategic discussion!

Now most interesting: $\sin^2 2\theta_{13} \sim 10^{-2} - 10^{-1}$
- Later decisions based on results in this range
- New technologies will change discussion

We discussed a logarithmic scale of $\sin^2 2\theta_{13}$
- Is this an appropriate representation?
- Linear scale from linear mass models in flavor space?