Supersymmetric Electroweak Baryogenesis and Mixing in Transport Equations

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Outline

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- EWB in the MSSM
- Former Approaches
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Introduction

Baryogenesis is one of the cornerstones of the Cosmological Standard Model.

The celebrated Sakharov conditions state the necessary ingredients for baryogenesis:

- C and CP violation
- non-equilibrium
- B-violation

B-violation is in the hot universe present due to sphaleron precesses.

C is violated in the electroweak sector of the SM.
The sphaleron process:

- $\Delta B = 3$, $\Delta L = 3$, $\Delta N_{CS} = 1$
- $B + L$ violating
- $B - L$ conserving
- Exponentially suppressed by the $W$ mass
- Topological effect of the SU(2) gauge sector
Cross-over *versus* electroweak phase transition:

**Phase transition**

\[ V(h) \text{ at } T \sim 100 \text{ GeV} \]

\[ V(h) \text{ at } T \sim 100 \text{ GeV} \]

**Cross-over**

\[ V(h) \text{ at } T = 0 \text{ GeV} \]

\[ V(h) \text{ at } T \gg 100 \text{ GeV} \]
Introduction

Electroweak Baryogenesis:

Shaposhnikov ('87)
EWB in the MSSM

EWB in the SM excluded:

- CP violation too small
- No strong first order phase transition

EWB in the MSSM not excluded/disproved:

- CP violation in the Higgsino/chargino sector
- Strong first order phase transition in case of a light stop
In the MSSM case the Higgsino/chargino mass matrix is:

\[ m = \begin{pmatrix} M_2 & g h_2(z) \\ g h_1(z) & \mu_c \end{pmatrix} \]

with \( M_2 \) and \( \mu_c \) complex and

\[ \partial_z m = \begin{pmatrix} 0 & g \partial_z h_2(z) \\ g \partial_z h_1(z) & 0 \end{pmatrix} \]

and \( \text{Tr} \left( \partial_z M M^\dagger - M \partial_z M^\dagger \right) \sim \text{Im}(M_2 \mu_c^*) \neq 0 \). CP-violation is present on the tree level while in the SM \( \partial_z M \sim M \).
Connection between macroscopic and microscopic scales:

- CP violation is a microscopic effect produced by interaction of single quanta in the plasma.
- Transport is a macroscopic effect based on statistical physics and particle densities.

Filling this gap means deriving semiclassical Boltzmann equations from first principles.
A) Via dispersion relations:

**Cline, Joyce, Kainulainen (’97, ’00)**

The CP violating dispersion relation $E(p, z)$ for a varying mass $m(z) = |m(z)|e^{i\theta(z)}$ is determined by the WKB method and put into the classical Boltzmann equation for the particle density $f$:

$$E^2 = \vec{p}^2 + m^2 \pm \frac{m^2 \partial_z \theta}{2p_z}$$

$$(\partial_t + \partial_{p_z} E \partial_z - \partial_z E \partial_{p_z}) f = \text{Coll.}$$

This procedure does not contain mixing effects.
Former Approaches

B) Via source terms:

\textbf{C}ARENA, \textbf{M}ORENO, \textbf{Q}UIROS, \textbf{S}ECO, \textbf{W}AGNER (’00, ’02)

The additional term in the Schwinger-Dyson equation is interpreted as an interaction term

\[ j^\mu(X_\mu) = \frac{1}{\tau} \int \frac{d^4p}{(2\pi)^4} \ p^\mu \delta g(p_\mu, X_\mu) \]

and put 'by hand' into diffusion equations.
c) Kadanoff-Baym Equations:

Statistical analogon to Schwinger-Dyson equations

\[
(S_0^{-1} - \Sigma_R) S^< - \Sigma^< S_R = \frac{1}{2} \Sigma^< S^> - \frac{1}{2} \Sigma^> S^< = \text{Coll.},
\]

\[
(S_0^{-1} - \Sigma_R) \mathcal{A} - \Sigma_A S_R = 0,
\]

and all functions depend on \( x_\mu \) and \( y_\mu \). Used definitions:

\[
S^< := S^{+-}, \quad S^> := S^{-+},
\]

\[
\mathcal{A} := \frac{i}{2}(S^< - S^>), \quad S_R := S^{++} - \frac{i}{2}(S^< + S^>).
\]
Wigner space: Using

\[ X = \frac{(x + y)}{2}, \quad r = \frac{(x - y)}{2} \]

and performing a Fourier transformation with respect to \( r \)

\[
e^{-i\hat{\phi}} \{ S_0^{-1} - \Sigma_R, S^< \} - e^{-i\hat{\phi}} \{ \Sigma^<, S_R \} = \text{Coll.},
\]

\[
e^{-i\hat{\phi}} \{ S_0^{-1} - \Sigma_R, \mathcal{A} \} - e^{-i\hat{\phi}} \{ \Sigma_A, S_R \} = 0,
\]

with

\[ 2\hat{\phi}\{ A, B \} := \partial_{X^\mu} A \partial_{k^\mu} B - \partial_{k^\mu} A \partial_{X^\mu} B, \]

all quantities become functions of \( X^\mu \) and \( k^\mu \).
Free bosonic theory with a constant mass in equilibrium:

- The constraint and kinetic equations are (KMS):
  \[
  (k^2 - m^2) S^< = 0, \quad k^\mu \partial_\mu S^< = 0
  \]

- And the solution reads:
  \[
  iS^< = 2\pi \text{sign}(k_0) \delta(k^2 - m^2) n(k_0), \quad n(k_0) = \frac{1}{\exp(\beta k_0) - 1}
  \]

where we can read off the particle density

\[
\int_{k_0 > 0} \frac{d^4 k}{(2\pi)^4} 2ik_0 S^< = \int \frac{d^3 k}{(2\pi)^3} n(\sqrt{k^2 + m^2})
\]
Gradient expansion:

For space-time dependent mass matrices, the Kadanoff-Baym equations can be expanded in gradients as long as the background is weakly varying

\[
\frac{\partial_k \partial_X}{1} \ll 1 \quad \rightarrow \quad e^{-i\diamond} \approx 1 - i\diamond
\]

what is true in the MSSM \((l_w \approx 20/T_c)\).

The simplest example for an transport equation in a varying background is for one flavour with real mass

\[
(k^2 - m^2)S^< = 0, \quad (k^\mu \partial_\mu - \frac{1}{2} (\partial_z m^2) \partial_{k_z})S^< = \text{Coll}.
\]
After spin projection and in the wall frame:

\[
\begin{align*}
2i k_0 g_0^s - (2i s k_z + s \partial_z) g_3^s - 2i m_h \hat{E} g_1^s - 2i m_a \hat{E} g_2^s &= 0 \\
2i k_0 g_1^s - (2s k_z - is \partial_z) g_2^s - 2i m_h \hat{E} g_0^s + 2m_a \hat{E} g_3^s &= 0 \\
2i k_0 g_2^s + (2s k_z - is \partial_z) g_1^s - 2m_h \hat{E} g_3^s - 2i m_a \hat{E} g_0^s &= 0 \\
2i k_0 g_3^s - (2i s k_z + s \partial_z) g_0^s + 2m_h \hat{E} g_2^s - 2m_a \hat{E} g_1^s &= 0
\end{align*}
\]

with

\[
\hat{E} \equiv \exp \left( \frac{i}{2} \frac{\partial}{\partial z} \frac{\partial}{\partial k_z} \right), \quad \hat{E}^\dagger \equiv \exp \left( -\frac{i}{2} \frac{\partial}{\partial k_z} \frac{\partial}{\partial z} \right).
\]
In first order the kinetic equation is: \( g = g^{eq} + \delta g \)

\[
k_z \partial_z \delta g_R + \frac{i}{2} \left[ m^\dagger m, \delta g_R \right] - \frac{1}{4} \left\{ (m^\dagger m)', \partial_{k_z} \delta g_R \right\}^D + k_0 \Gamma \delta g_R - k_z \left[ V^\dagger \nu, \delta g_R \right] = S_R \[ g^{eq}_R, g^{eq}_L \]
\]

- The source \( S \) depends on the lowest order solutions \( g^{eq}_R \) and \( g^{eq}_L \)
- We introduced a damping term in the time-like direction to fulfill the boundary conditions at \( z \to \pm \infty \) simultaneously
Sources:

\[ S_R[g_R^{eq}, g_L^{eq}] = -k_z \left[ V^\dagger V, \hat{g}_0^{eq} - \hat{g}_3^{eq} \right] + \frac{1}{4} \left\{ (m^\dagger m)', \partial_{k_z} (\hat{g}_0^{eq} - \hat{g}_3^{eq}) \right\}^T \]

\[-\frac{s}{4k_0} \left[ m^\dagger m - m^\dagger m', \hat{g}_0^{(0)} \right]^T - \frac{s}{4k_0} \left\{ (m^\dagger m)', \hat{g}_0^{(0)} \right\}^T \]

where

\[ \hat{g}_3^{eq} \equiv V^\dagger g_3^{eq} V = (g_R^{(0)} - V^\dagger U g_L^{eq} U^\dagger V) / 2 \]

\[ \hat{g}_0^{eq} \equiv V^\dagger g_0^{eq} V = (g_R^{(0)} + V^\dagger U g_L^{eq} U^\dagger V) / 2 \]
Comments:

- The constraint equation is non-algebraic.
- It is possible to decouple the kinetic equations, such that it does not contain $k_0$ explicitly and no information about the spectral function is needed.
- The kinetic equation for the deviation from thermal equilibrium in the mass eigenbasis reads

\[ g = g^{eq} + \delta g \]

\[ k_z \partial_z \delta g + \frac{i}{2} \left[ m^2, \delta g \right] + \cdots = S(g^{eq}) \]
The term $\frac{i}{2} \left[ m^2, \delta g \right]$ will lead to an oscillatory behaviour of the off-diagonal particle densities, similar to neutrino oscillations with frequency $\sim \frac{(m_1^2 - m_2^2)}{k_z}$. This can be seen using

$$i \left[ \sigma_3, \sigma_1 \right] = \sigma_2, \quad i \left[ \sigma_3, \sigma_2 \right] = -\sigma_1$$

Without this oscillation term CP would be conserved up to first order in gradients.

The phenomenological damping term $k_0 \Gamma \delta g$ has been introduced \textit{after} identification of CP violating sources.
Parameters chosen: $M_2 = 200$ GeV, $\mu_c = 220$ Gev
Concluding remarks:

The first principle derivation of the transport equations shows:

- Oscillations in the CP violating densities are important for the dynamics
- CP violation on the microscopic level has to be identified before time-invariance is broken by phenomenological damping terms

and both effects suppress CP-violating effects relative to former approaches.
Diffusion and the Sphaleron

Missing parts to determine the baryon asymmetry of the universe:

and
Diffusion equations:

\[ v_w n_Q' = D_q n_Q'' - \Gamma_Y \left[ \frac{n_Q}{k_Q} - \frac{n_T}{k_T} - \frac{n_H + n_h}{k_H} \right] - \Gamma_m \left[ \frac{n_Q}{k_Q} - \frac{n_T}{k_T} \right] \]

\[ -6 \Gamma_{ss} \left[ 2 \frac{n_Q}{k_Q} - \frac{n_T}{k_T} + 9 \frac{n_Q + n_T}{k_B} \right] \]

\[ v_w n_T' = D_q n_T'' + \Gamma_Y \left[ \frac{n_Q}{k_Q} - \frac{n_T}{k_T} - \frac{n_H + n_h}{k_H} \right] + \Gamma_m \left[ \frac{n_Q}{k_Q} - \frac{n_T}{k_T} \right] \]

\[ + 3 \Gamma_{ss} \left[ 2 \frac{n_Q}{k_Q} - \frac{n_T}{k_T} + 9 \frac{n_Q + n_T}{k_B} \right] \]
Weak sphaleron:

\[ n_B = - \frac{3 \Gamma_{ws}}{v_w} \int_{-\infty}^{0} dz \, n_L(z) \exp \left( \frac{z}{4\Gamma_{ws}} \frac{15\Gamma_{ws}}{v_w} \right) \]

and finally the baryon-to-entropy ratio is determined via

\[ \eta = \frac{n_B}{s}, \quad s \approx 51.1 \, T_c^3 \]

and can be compared with

\[ \eta_{BBN} \approx 0.9 \times 10^{-10} \]
Numerical Results

Parameters chosen: $M_2 = 200$ GeV, $v_w = 0.05$, $l_w = 20/T_c$, $m_A = 200$ GeV, $\sin(\phi) = 1.0$
Numerical Results

Parameters chosen: $v_w = 0.05$, $l_w = 20/T_c$, $m_A = 200$ GeV
Pessimistic Conclusion

Supersymmetric baryogenesis is a rather unlikely scenario based on

- A light stop to acquire a strong first order phase transition
- The condition $\mu_c \approx M_2 \ll 400$ GeV of the a priori unrelated parameters $M_2$ and $\mu_c$
- A large CP-violating phase that hardly satisfies experimental EDM bounds
We have been able to derive a formalism that reproduces the qualitative behaviour of two existing methods in their range of applicability. In addition we found two additional features in the transport equations not taken into account before.

Based on our analysis, supersymmetric baryogenesis is a scenario that

- Can explain the BAU
- Predicts $\mu_c \approx M_2 \ll 400 \text{ GeV}$
- Will be testable soon
The End