Simulation of neutrino oscillations including non-standard effects using the MINOS experiment

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Outline

- Neutrino oscillations
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- Neutrino oscillations
- Non-standard interactions in neutrino oscillations
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- Non-standard interactions in neutrino oscillations
- The MINOS experiment
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- Neutrino oscillations
- Non-standard interactions in neutrino oscillations
- The MINOS experiment
- Outcome of simulations
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- Non-standard interactions in neutrino oscillations
- The MINOS experiment
- Outcome of simulations
- Summary
Charged leptons and neutrinos

There are three charged leptons

- electron $e$
- muon $\mu$
- tau $\tau$
Charged leptons and neutrinos

There are three charged leptons

- electron $e$
- muon $\mu$
- tau $\tau$

and three corresponding neutrinos

- electron neutrino $\nu_e$
- muon neutrino $\nu_{\mu}$
- tau neutrino $\nu_{\tau}$
Neutrino mixing

- The leptonic mixing matrix $U$ is the leptonic analogue of the Cabibbo–Kobayashi–Maskawa (CKM) matrix in the quark sector.
Neutrino mixing

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- Two-flavour case

\[
U = \begin{pmatrix}
\cos \theta_0 & \sin \theta_0 \\
-\sin \theta_0 & \cos \theta_0
\end{pmatrix}
\]
Neutrino mixing

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- Three-flavour case
  - ...much more complicated
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$$U = \begin{pmatrix} \cos \theta_0 & \sin \theta_0 \\ -\sin \theta_0 & \cos \theta_0 \end{pmatrix}$$

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  - ... much more complicated
  - $U$ depends on three mixing angles $\theta_{12}$, $\theta_{13}$ and $\theta_{23}$ and one CP-violating phase $\delta$. 

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Neutrino oscillations

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Neutrino oscillations

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- For example: two-flavour case in vacuum

$$P_{e\mu} = P_{\mu e} = \sin^2(2\theta_0) \sin^2 \left( \frac{\Delta m^2}{4E} L \right)$$

$$\Delta m^2 = m_2^2 - m_1^2$$
Neutrino oscillations

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P_{e\mu} = P_{\mu e} = \sin^2(2\theta_0) \sin^2\left(\frac{\Delta m^2}{4E} L\right)
\]

\[
P_{ee} = P_{\mu\mu} = 1 - \sin^2(2\theta_0) \sin^2\left(\frac{\Delta m^2}{4E} L\right)
\]

\[
\Delta m^2 = m_2^2 - m_1^2
\]
Neutrino oscillations

- Three-flavour case in vacuum

\[
P_{e\mu} = P_{\mu e} = \sin^2(2\theta_{13}) \sin^2(\theta_{23}) \sin^2 \left( \frac{\Delta m^2_{32}}{4E} L \right)
\]

\[
\Delta m^2_{32} = m_3^2 - m_2^2 \\
\Delta m^2_{21} = m_2^2 - m_1^2 \to 0
\]
Neutrino oscillations

- Three-flavour case in vacuum

\[ P_{e\mu} = P_{\mu e} = \sin^2(2\theta_{13}) \sin^2(\theta_{23}) \sin^2 \left( \frac{\Delta m_{32}^2}{4E} L \right) \]

\[ P_{e\tau} = P_{\tau e} = \sin^2(2\theta_{13}) \cos^2(\theta_{23}) \sin^2 \left( \frac{\Delta m_{32}^2}{4E} L \right) \]

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Neutrino oscillations

- Neutrino scattering in matter (coherent forward scattering)
Neutrino oscillations

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- NC-interactions contribute with an overall phase
Neutrino oscillations

- Neutrino scattering in matter (coherent forward scattering)

- NC-interactions contribute with an overall phase
- Only CC-interactions give an effective contribution
Neutrino oscillations

- Effective Hamiltonian in matter

\[ H_{\text{fl}} = \frac{1}{2E} U \begin{pmatrix} 0 & 0 & 0 \\ 0 & \Delta m_{21}^2 & 0 \\ 0 & 0 & \Delta m_{31}^2 \end{pmatrix} U^\dagger + \begin{pmatrix} V_{CC} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \]
Neutrino oscillations

- Effective Hamiltonian in matter

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- \( V_{CC} = \sqrt{2} G_F N_e \) is the effective matter potential
Neutrino oscillations

- There could be other interactions between neutrinos and fermions in matter (i.e. $e$, $u$ and $d$) which affect neutrino oscillations.
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Neutrino oscillations

- There could be other interactions between neutrinos and fermions in matter (i.e. $e$, $u$ and $d$) which affect neutrino oscillations
- $\Rightarrow$ Non-standard interactions (NSI)
- Effective Hamiltonian in matter including NSI

$$H_{fl} = \frac{1}{2E} U \begin{pmatrix} 0 & 0 & 0 \\ 0 & \Delta m_{21}^2 & 0 \\ 0 & 0 & \Delta m_{31}^2 \end{pmatrix} U^\dagger$$

$$+ V_{CC} \begin{pmatrix} 1 + \epsilon_{ee} & \epsilon_{e\mu} & \epsilon_{e\tau} \\ \epsilon_{e\mu}^* & \epsilon_{\mu\mu} & \epsilon_{\mu\tau} \\ \epsilon_{e\tau}^* & \epsilon_{\mu\tau}^* & \epsilon_{\tau\tau} \end{pmatrix}$$
Neutrino oscillations

- There could be other interactions between neutrinos and fermions in matter (i.e. $e$, $u$ and $d$) which affect neutrino oscillations
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- Effective NSI parameters $\epsilon_{\alpha\beta}$
Neutrino oscillations

- Present bounds on NSI parameters
Neutrino oscillations

- Present bounds on NSI parameters

| $|\epsilon_{ee}| \sim O(1)$ | $|\epsilon_{e\mu}| < 0.010$ | $|\epsilon_{e\tau}| \sim O(1)$ |
|--------------------------|-------------------|--------------------------|
| $|\epsilon_{\mu\mu}| < 0.017$ | $|\epsilon_{\mu\tau}| < 0.013$ | $|\epsilon_{\tau\tau}| \sim O(1)$ |

S. Davidson et al., JHEP 03, 011 (2003), hep-ph/0302093
Neutrino oscillations

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- $\Rightarrow \epsilon_{e\mu}, \epsilon_{\mu\mu}$ and $\epsilon_{\mu\tau}$ can be neglected
**Neutrino oscillations**

- Present bounds on NSI parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Bound</th>
</tr>
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<tbody>
<tr>
<td>$</td>
<td>\epsilon_{ee}</td>
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<td>$</td>
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- $\Rightarrow \epsilon_{e\mu}, \epsilon_{\mu\mu}$ and $\epsilon_{\mu\tau}$ can be neglected

- What constraints could be put on $\epsilon_{ee}, \epsilon_{e\tau}$ and $\epsilon_{\tau\tau}$ by future experiments?
The MINOS experiment

- Main Injector Neutrino Oscillation Search (MINOS)
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- Long-baseline neutrino experiment (735 km)
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- Far Detector at Soudan mine, northern Minnesota
- Takes data since 2005, preliminary results in summer 2006 after approximately one year of running

The MINOS experiment

- Geographical layout
Oscillation channels at MINOS

- Disappearance channel
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  - $\nu_\mu$ oscillates mainly into $\nu_\tau$, but also into $\nu_e$
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  - Sensitive to $\theta_{13}$ and $\epsilon_{e\tau}$
MINOS experiment simulation

- Simulated with the General Long Baseline Experiment Simulator (GLoBES)
  
MINOS experiment simulation

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- Simulation of five years running time
MINOS experiment simulation

- Simulated with the General Long Baseline Experiment Simulator (GLoBES)

- Simulation of five years running time

- Neutrino oscillation and NSI parameters used for the simulations

\[
\begin{align*}
\sin^2(2\theta_{12}) &= 0.8 \\
\sin^2(2\theta_{13}) &= 0.07 \text{ or } 0 \\
\sin^2(2\theta_{23}) &= 1 \\
\Delta m_{21}^2 &= (7 \cdot 10^{-5}) \text{ eV}^2 \\
\Delta m_{32}^2 &= (2.74 \cdot 10^{-3}) \text{ eV}^2 \\
\delta &= \frac{\pi}{2}
\end{align*}
\]

\[
\begin{align*}
\epsilon_{ee} &= 0 \\
\epsilon_{e\mu} &= 0 \\
\epsilon_{e\tau} &= 0 \\
\epsilon_{\mu\mu} &= 0 \\
\epsilon_{\mu\tau} &= 0 \\
\epsilon_{\tau\tau} &= 0
\end{align*}
\]
MINOS experiment simulation

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- NSI parameters are assumed to be real
Simulation results

- $\sin^2(2\theta_{23}) - \Delta m_{32}^2$ plane

![Graph showing the $\sin^2(2\theta_{23}) - \Delta m_{32}^2$ plane with contours for different confidence levels with and without NSI, including a best-fit point for both channels.]
Simulation results

- NSI effects on $\sin^2(2\theta_{23})$
Simulation results

- NSI effects on $\Delta m^2_{32}$
Simulation results

- Constraints on NSI parameters

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\sin^2(2\theta_{13}) = 0.07
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Simulation results

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\]
Simulation results

- Constraints on NSI parameters

| $\sin^2(2\theta_{13})$ | $-2.16 < \epsilon_{e\tau} < -1.31$
|------------------------|-------------------|
| $\sin^2(2\theta_{13}) = 0$ | $-0.60 < \epsilon_{e\tau} < 0.41$
| **Confidence level** | **90 %**

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Simulation results

- NSI effects on $\sin^2(2\theta_{13})$
Summary and Conclusion

- Allowed region in the $\sin^2(2\theta_{23}) - \Delta m_{32}^2$ plane is extended to lower values of $\sin^2(2\theta_{23})$ and to higher values of $\Delta m_{32}^2$ if NSI effects are present.
Summary and Conclusion

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- The reason for this extension is the absence of bounds on $\epsilon_{\tau\tau}$. 
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- Possible bounds on the NSI parameter $\epsilon_{e\tau}$ depending on the value of $\theta_{13}$. 
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- Value of $\sin^2(2\theta_{13})$ could be higher than the presently given bound if NSI effects are present.
Summary and Conclusion

- Allowed region in the \( \sin^2(2\theta_{23}) - \Delta m_{32}^2 \) plane is extended to lower values of \( \sin^2(2\theta_{23}) \) and to higher values of \( \Delta m_{32}^2 \) if NSI effects are present.
- The reason for this extension is the absence of bounds on \( \epsilon_{\tau \tau} \).
- Possible bounds on the NSI parameter \( \epsilon_{e\tau} \) depending on the value of \( \theta_{13} \).
- Value of \( \sin^2(2\theta_{13}) \) could be higher than the presently given bound if NSI effects are present.
- Future experiments should be able to put strict constraints on \( \epsilon_{e\tau} \) and \( \epsilon_{\tau \tau} \).