Neutrino Masses, Grand Unification and Extra Dimensions

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Presented at the
Introduction

Theoretical Challenges Posed by Neutrino Observations

➤ Why $m_\nu \ll m_{u,d,e}$?

➤ Why are neutrino mixings so much larger than quark mixings?

➤ How do neutrinos fit into the big picture of grand unification, extra dimensions etc. that relate to other particle physics issues?

➤ What are the neutrino results telling us about the nature of new physics beyond the standard model e.g. new symmetries, new forces, new particles etc.
Standard model

Details

- Gauge group $SU(2)_L \times U(1)_Y$

- Matter: Doublets: $Q \equiv \begin{pmatrix} u_L \\ d_L \end{pmatrix}$; $\psi_L \equiv \begin{pmatrix} \nu_L \\ e_L \end{pmatrix}$;
  Singlets: $u_R; \ d_R; \ e_R$

- Higgs: $H \equiv \begin{pmatrix} H^0 \\ H^- \end{pmatrix}$ with $\langle H^0 \rangle = v_{wk}$

- $\mathcal{L}_Y = h_u \bar{Q}_L H u_R + h_d \bar{Q}_L \tilde{H} d_R + h_e \bar{\psi}_L \tilde{H} e_R + h.c.$

- Gives $m_\nu = 0$
  : Reason $B - L$ is an exact symmetry;
  $m_\nu \neq 0$ should therefore be related to B-L symmetry breaking.
Does neutrino mass necessarily require physics beyond the standard model?

- **B-L breaking by gravity**
  - Global symmetries could be broken by nonperturbative gravitational effects such as black holes or worm holes etc.
  - If so, they could induce B-L breaking operators into standard model e.g. \( (\psi_L H)^2 / M_{Pl} \);
  - They lead to \( m_\nu \sim \frac{v_{weak}^2}{M_{Pl}} \sim 10^{-5} \text{ eV} \) clearly too small to explain atmospheric neutrino deficit.

- **New physics beyond std model required to understand** \( m_\nu \neq 0 \)

Akhmedov, Bereziani, and Šenjanovic
Std model successful but unsatisfactory

1. Not symmetric between quarks and leptons, even though weak interactions are

2. What is the origin of parity violation?

3. Electric charge formula: \( Q = I_{3L} + \frac{Y}{2} \);
   we know what is \( I_{3L} \); what is \( Y \)- an adjustable parameter!!

4. Can neutrinos help us understand these issues better?
Neutrino mass and Nature of new physics

Simplest possibility: Add $\nu_R$ to the standard model

- New term in the $\mathcal{L}_Y$: $h_\nu \bar{\psi}_L H \nu_R + h.c.$

- Since $\nu_R = N_R$ is std model singlet, new term allowed by gauge invariance: $M_R N_R^T C^{-1} N_R + h.c.$

  Important point: $M_R$ breaks B-L symmetry

- $\rightarrow$ Mass matrix for $(\nu_L, N_R)$ system:

\[
\begin{pmatrix}
0 & h_\nu v \\
h_\nu^T v & M_R
\end{pmatrix}
\]

- Since $M_R \gg h_\nu v$, mass eigenvalues have a heavy : $\rightarrow$: $M_R$ and a light set: $M_\nu \simeq -\frac{h_\nu^2 v^2}{M_R}$. This implies $m_{\nu_i} \ll m_{u,d,e}$...

- Seesaw mechanism $\rightarrow$ explains the smallness of neutrino mass

Gell-Mann, Ramond, Slansky; Yanagida; Glashow; R. N. M., Senjanovic (1979)
Implications of Seesaw

- Neutrino is Majorana: Can lead to neutrinoless double beta decay and other $\Delta L = 2$ processes;

- Scale of the RH neutrino mass: roughly speaking:
  \[ M_{R,max} \sim \frac{m^2_i}{\sqrt{\Delta m^2_A}} \sim 10^{14} - 10^{15} \text{ GeV} \]
  
  $M_R$ close to the conventional SUSY GUT scale!!

  Could $m_\nu$ be the first indication of grand unification?
Why Seesaw is theoretically so appealing?

Adding $N_R$ to std model makes fermion spectrum quark-lepton symmetric.

Makes the spectrum also left-right symmetric: under Parity

\[
\begin{pmatrix}
u_L \\ e_L
\end{pmatrix} \leftrightarrow \begin{pmatrix}
u_R \\ e_R
\end{pmatrix};
\begin{pmatrix} u_L \\ d_L
\end{pmatrix} \leftrightarrow \begin{pmatrix} u_R \\ d_R
\end{pmatrix};
\]

Electroweak gauge group expands to $SU(2)_L \times SU(2)_R \times U(1)_{B-L}$
i.e. weak interactions become parity conserving

\[
\mathcal{L}_{wk} = \frac{g}{2\sqrt{2}} (\vec{W}_{\mu,L} \cdot \vec{J}_L^\mu + \vec{W}_{\mu,R} \cdot \vec{J}_R^\mu)
\]

Pati, Salam (73); R. N. M., Pati (74); Senjanovic, R. N. M. (75)
Neutrino mass and parity violation

☞ Two Questions for Left-right models

1. Why are low energy weak int. V-A ?

2. Why \( m_\nu \ll m_{u,d,e} \)?

☞ Both questions have the same answer:

➤ BREAK PARITY AT SCALE MUCH ABOVE THE \( W_L \) MASS

\[
SU(2)_L \times SU(2)_R \times U(1)_{B-L} \rightarrow G_{std} \rightarrow U(1)_{em}
\]

\[
M_{\nu,N} = \begin{pmatrix} 0 & 0 \\ 0 & M_R \end{pmatrix} \rightarrow \begin{pmatrix} f v_L & h_{\nu} v \\ h_{\nu}^T v & f v_R \end{pmatrix} \text{(SEESAW)}
\]

➤ As before, \( m_\nu \approx f v_L - \frac{h_{\nu}^2 v^2}{f v_R} \); \( (v_L \sim \frac{v^2_{\nu k}}{v_R}) \).
Strength of V'+A currents \( \propto \frac{1}{v_R} \);
as the scale of parity violation \( v_R \rightarrow \infty, m_\nu \rightarrow 0 \);

➤ SMALLNESS OF \( m_\nu \) CONNECTED TO THE SUPPRESSION OF V'+A currents
Asymptotic parity and seesaw

- Lesson from LR models
  \[ m_\nu \simeq f \frac{v_{uk}^2}{v_R} - \frac{h^2 v_{uk}^2}{f v_R}; \text{(Type II seesaw)} \]
- Parity \(\rightarrow\) Type II seesaw

No parity symmetry

- \[ m_\nu \simeq -\frac{h^2 v_{uk}^2}{f v_R} \text{ (Type I seesaw)} \]

Type I diagram

Type II diagram

Lazaridis, Shafi and Wetterich; R. N. M. and Senjanovic (1980).
A simple pointer to Type II seesaw

➤ Since $h_\nu \sim h_e \sim$ hierarchical, Type I seesaw $\rightarrow$

\[ m_1 \ll m_2 \ll m_3 \] (hierarchical) i.e.

➤ Quasidegenerate neutrinos $\rightarrow$ prefers Type II seesaw
How to understand large mixings?

**Two generation example: atmospheric angle**

- Choose basis where $\mu, \tau$ are mass eigenstates

\[
\mathcal{M}_\nu = \begin{pmatrix} a & b \\ b & a \end{pmatrix} \rightarrow U_\nu = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ -1 & 1 \end{pmatrix}; \text{ Maximal mixing;}
\]

- Mass hierarchy i.e. $\Delta m^2_\odot \ll \Delta m^2_A \rightarrow (a - b) \ll (a + b)$

- Mass matrix for atmospheric angle and solar mass is:

\[
\mathcal{M}_\nu = \sqrt{\Delta m^2_A} \begin{pmatrix} 1 + \epsilon & 1 \\ 1 & 1 \end{pmatrix};
\]

- A lesson from large atmospheric mixing for theorists is: a possible $\nu_\mu \leftrightarrow \nu_\tau$ symmetry.

- HOW TO EXTEND IT TO THREE GENERATIONS and HOW TO TEST FOR THIS SYM.?
Three generation mass matrix consistent with large solar and near maximal atmospheric

Case of Normal Hierarchy

Choose basis where $e, \mu, \tau$ are mass eigenstates

\[ M_\nu = \sqrt{\Delta m^2_A} \begin{pmatrix} d \epsilon^n & b \epsilon & a \epsilon \\ b \epsilon & 1 + \epsilon & 1 \\ a \epsilon & 1 & 1 + c \epsilon \end{pmatrix}; \ n \geq 1. \]

Leads to both large $\theta_{23}$ and large $\theta_{12}$ and small $\theta_{13}$.

Determines $\epsilon \sim \sqrt{\frac{\Delta m^2_{\odot}}{\Delta m^2_A}} \sim \theta_{\text{Cabibbo}} \sim \frac{1}{5}$

How can we experimentally determine the rest of the parameters? $\beta_\beta_{0\nu}$ measures $d$; unfortunately not very well for normal hierarchy WHAT ABOUT THE REST?
\( \theta_{13} \) can provide very important information

\( \theta_{13} \) probes \( \mu \leftrightarrow \tau \) symmetry and provides information about \( a, b, c \) in \( \mathcal{M}_V \): Three cases

\( \gg \) Exact \( \mu \leftrightarrow \tau \rightarrow a = b \) and \( c = 1 \rightarrow \theta_{13} = 0 \)

\( \gg \) Approximate \( \mu \leftrightarrow \tau \) symmetry \( a = b \) but \( c \neq 1 \rightarrow \)
\[ \theta_{13} \sim \epsilon^2 \sim \frac{\Delta m_{31}^2}{\Delta m_{32}^2} \sim 0.04; \]

\( \gg \) Hardly \( \mu \leftrightarrow \tau \) symmetry \( \rightarrow a, b \) arbitrary, \( c = 1 \) predicts
\[ \theta_{13} \sim \epsilon \sim \sqrt{\frac{\Delta m_{31}^2}{\Delta m_{32}^2}} \sim 0.2; \]

\( \gg \) knowledge of \( \theta_{13} \) important to understand the nature new symmetries for leptons.
Neutrino Mass and Grand unification

Grand unification of Forces is an attractive hypothesis

➢ Unifies three gauge couplings $g_s, g_2, g_1$ into one coupling at a high scale

GUT theories unifies quarks and leptons

1. Raises the hope of explaining the free parameters of the standard model

2. Solving gauge hierarchy needs supersymmetry and scale $M_U \sim 2 \times 10^{16}$ GeV
**Note that** $M_R \sim M_U$

- raises the hope that seesaw scale and GUT scale have common origin

- Perhaps neutrino masses and mixings can be predicted due to higher symmetry of GUT theories

**GUT needs SUSY to stabilize gauge hierarchy**

- SUSY pairs each fermion with a boson and vice versa: $Q \leftrightarrow \tilde{Q}$, $H \leftrightarrow \tilde{H}$, ...

- lightest SUSY particle $\tilde{H}$ can be dark matter $\rightarrow$ This requires SUSY model to be invariant under a new symmetry called R-parity

- MSSM does not have R-parity- RP inv. clue to SUSY model building beyond MSSM; we demand this as we build SUSY GUTs for neutrinos
SO(10) SUSY GUT just right for neutrinos

unification of all 16 fermions of one generation

\[
\begin{pmatrix}
  u & u & u & \nu \\
  d & d & d & e
\end{pmatrix}_{L,R}
\]

into 16 dim. rep of SO(10)

Contains the $N_R$ needed for seesaw automatically

Contains the left-right subgroup and the attractive properties of asymptotic parity conservation.

Contains the B-L subgroup
Breaking SO(10) down

- (i) \( \text{SO}(10) \rightarrow \text{SU}(5) \rightarrow \text{std model} \) or
- (ii) \( \text{SO}(10) \rightarrow SU(2)_L \times SU(2)_R \times SU(4)_C \rightarrow \text{std model} \)

- In either case one must break B-L symmetry
- How one breaks B-L leads to different physics:

Breaking B-L: 16 Higgs vrs 126 Higgs

- (a) 16- Higgs breaking \( \rightarrow \) no dark matter without additional assumptions
- (b) 126 Higgs breaking B-L leads automatically to dark matter: no additional symmetry needed

- Example of a theory with 126 breaking B-L
Minimal SUSY SO(10) For Neutrinos

Babu, RNM (92); Bajc, Senjanovic, Vissani (2002); Goh, RNM, Ng (03)

➢ Use only $\bf{126}$ to break B-L

➢ Some details: $\psi_a \bf{16}$- matter field; Higgs $\bf{10}(H), \bf{126}(\Delta) \oplus \bf{126}(\bar{\Delta}), \bf{210}$ (only first two couple to matter by group theory)

➢ Yukawa coupling

$$\mathcal{L}_Y = h_{ab} \psi_a \psi_b H + f_{ab} \psi_a \psi_b \bar{\Delta}$$

➢ Counting of parameters- 3 from $H, 6$ from $f$ plus 4 vevs minus $M_Z \rightarrow$ total of 12 parameters (no CP in Yukawas);

➢ Compare with standard model: vrs $10$ (quarks) + $3$ ($e, \mu, \tau$) + 18 for seesaw (total of 31).

➢ Input: masses of $(e, \mu, \tau)$, six quarks plus three CKM angles; All parameters of the fermion sector are determined; hence all but one neutrino masses and mixing angles predicted
Understanding large mixings without $\mu \leftrightarrow \tau$ or any other symmetry

- Key formula: $\mathcal{M}_\nu \simeq 10^{-9}(M_d - M_\ell)$ using type II seesaw
  
- gives $\mathcal{M}_\nu = m_b \begin{pmatrix} \lambda^4 & \lambda^4 & \lambda^3 \\ \lambda^4 & \lambda^2 & \lambda^2 \\ \lambda^3 & \lambda^2 & (1 - m_\tau/m_b) \end{pmatrix}$
  $\lambda \simeq 0.22$.

- Important point to note is that at the GUT scale $m_b \simeq m_\tau (1 + O(\epsilon))$

  For instance, for $\tan\beta = 10$,
  $m_b(M_U)/m_b(M_Z) \simeq 1^{+0.14}_{-0.08}$ and
  $m_\tau(M_U)/m_\tau(M_Z) \simeq 1.292 \pm 0.001$ (e.g. Das, Parida, 2002)

- This leads to large mixings $\theta_{23}$ and $\theta_{12}$ and small $\theta_{13}$ (close to the present upper limit);

- Furthermore, $\frac{\Delta m^2_{\odot}}{\Delta m^2_A} \sim \lambda^2 \gg (m_\mu/m_\tau)^2$ as required by data
Figure 1: Scatter corresponds to different allowed quark mass values

Figure 2: Scatter corresponds to uncertainty in quark mass values
Figure 3: $U_{e3} \equiv \theta_{13}$ and just below the present upper limit: “high” value due to quark lepton symmetry but no $\mu \leftrightarrow \tau$ symmetry (see before)

**SO(10) contd**

Testing Minimal SO(10) model for neutrinos

1. can be tested by Long Baseline experiments looking for $\theta_{13}$:

2. no signal in next gen. of $\beta\beta_{0\nu}$

3. predictions for proton decay: upper limits on $n \rightarrow K^0 \bar{\nu}$ and $p \rightarrow \pi^+ \bar{\nu}$ at the level of $10^{33}$ yrs;

4. CP violation by adding higher dimensional operators that transform effectively as $120$: model still predictive for $\theta_A$ and $\theta_{13} \geq 0.1$

(Dutta, Mimura, RNM, hep-ph/0406262)
Prediction of model with effective 120 operator
Figure 4: scatter corresponds to different allowed quark mass values

**Prediction of model with effective 120 operator**
Neutrino Mass and Large Extra Dimensions

A string inspired picture for space-time:

\[
\begin{array}{c}
\text{Bulk} \\
\nu_R(x, y) \quad y \rightarrow
\end{array}
\]

- \[ -\frac{R}{2} \leq y \leq \frac{R}{2}; \quad R \sim \text{millimeters} \]

Small \( m_\nu \) from Large Extra Dim.(2)

- \[ \mathcal{L} = \frac{1}{M_*} \int \delta^2(y) d^4 x d^2 y \bar{\psi}_L(x) H(x) \nu_R(x, y) + h.c. \]
- \( m_\nu \) is overlap probability of \( \nu_R \) at the brane \( P_{\nu_R} \) times weak scale
- \( P_{\nu_R}(y = 0) \approx \frac{1}{RM_*} \) (\( M_* \) is the width of the brane)
- \( m_\nu \approx \frac{\nu_{\text{weak}}}{RM_*} \)
- \( M_* R \approx \frac{M_{\text{Planck}}}{M_*} \approx 10^{14} \)
- \( m_\nu \approx 10^{-3} \text{ eV} \)
Picture of neutrino

Simple picture

- Mass of the bulk neutrinos: $m_n = \frac{n}{R}$ and
- Mixing with active neutrinos ($\nu_{e,\mu,\tau}$, $\xi_n \sim \frac{m_{\nu} R}{n}$)
- Other effects e.g. Magnetic moment connects $\nu_e - \nu_{n,R}$

\[ \mu_{en} \sim 10^{-19} \mu_B \frac{m_{\nu}}{1\text{ eV}} \]
Minimal Model and Oscillation data

Number of parameters

- Three bulk neutrinos
- Neutrinos Dirac type (assumption)
- number of parameters: 3 masses, three mixing angles: $\theta_{ij}$ and $\xi_i \simeq \sqrt{2}m_iR$
- Seven parameters in all
- Fitting oscillation data with usual large mixing solution possible
- SNO NC data $\rightarrow \xi_i \ll 1$

RNM, Perez-Lorenzana; Davoudiasl, Langacker, Perelstein; Barbieri, Creminelli and Strumia:
Manifestations of extra dimensions

- MSW effects of KK tower of bulk neutrinos

\[ \beta = \frac{G_F E R^2 \rho}{\sqrt{2M_N a_F}}; \text{ Resonances for } \beta = 1, 2, 3, 4 \]

- Solar Neutrinos: \( E \sim 10 \text{ MeV}; R \sim 50 \mu m \) (first resonance)
  UHE Neutrinos: \( E \sim 100 \text{ GeV}; R \sim 1 \mu m \) (using \( \rho \sim 1 \text{ gram/cm}^3 \))

- At resonance, survival probability: \( P \simeq e^{-\frac{2\pi m^2 R^2 \xi^2 R_{eff}}{E}} \)
  for \( E \geq E_{res} \) and \( \sim 1 \) otherwise

Leads to Dip structure in \( P \) as a function of \( E \);

\( (\text{Dvali,Smirnov}) \)

- no dip structure in solar spectrum \( \rightarrow R \leq 20 \mu m \).
An example of dip structure in solar neutrinos

Figure 5: Energy dependence of the $\nu_e$ survival probability when $R = 58 \mu m$, $mR = 0.0093$, $\delta_{\nu e} = 0.84 \times 10^{-7}$ eV. The dot-dashed part of the curve assumes the radial dependence in the Sun for neutrinos from the pp reaction, the solid part assumes $^{15}O$ radial dependence, and the dashed part assumes $^8B$ radial dependence.

Caldwell, RNM. Yellin, 2001
Spin-flip effect

An interesting effect occurs for magnetic moment: active neutrino has identical $\mu_{en}$ to all KK tower levels; $\mu_{en} \simeq 10^{-19}(m_n/1\,\text{eV})$.

In $\nu - e$ scattering, all KK modes upto $E_\nu$ contribute, enhancing the effect.

Figure 6: The figure presents the folded differential cross section $\frac{d\sigma}{dT}$ for the magnetic moment contribution to $\bar{\nu}e$ scattering for the cases of $D = 0$, $D = 2$, $D = 3$ and the standard weak contribution. The case $D = 0$ corresponds to the standard model extended by the inclusion of one right handed neutrino per family.
Seesaw vrs Large extra dimensions

1. Naturalness issue: why is \( \frac{LHLH}{M} \) small? for seesaw \( M \approx M_U \); so no problem- but for LED models, \( M \approx \text{TeV} \) and hence this effect is huge. Problem for Large Extra dim models

2. Seesaw Physics only indirectly accessible (via lepton flavor violation etc) where LED physics directly accessible to colliders.

3. More complete theories available for seesaw models but not for LED models.

4. Seesaw models provide a nice framework for origin of matter- origin of matter a complicated issue for LED models

☞ My personal bias: View extra dimensional effects discussed as non-standard neutrino physics that can shed light on a particular class of large extra D models
Conclusions

➢ Seesaw is the simplest way to understand small neutrino masses;

➢ it strongly suggest GUT theories with SO(10) as the prime candidate;

➢ SO(10) generically predicts Normal hierarchy (or degenerate with additional assumptions) but not inverted; sign of $\Delta m^2_{31}$ will be important for generic simple SO(10) models.

➢ A minimal SUSY SO(10) is very predictive in the neutrino sector and testable via the measurement of $\theta_{13}$. 
Extra dimension models provide a completely different way to understand small neutrino masses, but suffer from naturalness problems.

View this as a source of novel nonstandard neutrino physics.

leading to dip structure in survival probabilities when $\nu$ traverses through matter, enhanced cross-section for neutrino scattering at low and high energies, which could be looked for in experiments such as reactor searches for neutrino magnetic moment etc.
As we enter the era of Precision Neutrino Measurement Science (PNMS era), we will not only elevate our knowledge of neutrinos to the level of quarks but we will also learn a great deal about the nature of new physics (new forces, new particles, new mass scales etc) beyond the standard model.