

# Neutrino oscillations and time-energy uncertainty relation

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## K2K and MINOS accelerator experiments

important confirmation of the phenomenon of  
neutrino oscillations

discovered and studied by the SK, SNO,  
KamLAND and other experiments

K2K and MINOS are oscillation experiments  
of a new type

For the first time the time of neutrino  
production and detection was measured

In the K2K experiment neutrinos were  
produced in 1.1  $\mu\text{sec}$  spills

$\Delta t = t_{SK} - t_{KEK}$  was measured

Selected events satisfy the criteria

$$-0.2 \leq (\Delta t - t_{TOF}) \leq 1.3 \mu\text{sec}$$

$$t_{TOF} = L/c \simeq 0.83 \cdot 10^3 \mu\text{sec}$$

proved that  $\Delta t \simeq L$  ( $L \simeq 250$  km)

These new oscillation experiments are important for the understanding of the physical origin of neutrino oscillations

We must take into account that neutrino oscillations is phenomenon with finite  $\Delta t$  during which neutrino state significantly changed

For such phenomena time-energy uncertainty relation

$$\Delta E \Delta t \geq 1$$

This means that neutrinos must be described by non stationary states

Neutrino oscillations and  $B_d \rightarrow \bar{B}_d$  etc have the same physical origin (B. Pontecorvo, 1958)

Must be treated in the same way

A lot of new high-precision B-factory data

Perfectly described by Schrodinger equation  
and non stationary states

## Quantum field theoretical basics

I. Standard Model interaction with currents

$$j_{\alpha}^{\text{CC}} = 2 \sum_{l=e,\mu,\tau} \bar{\nu}_{lL} \gamma_{\alpha} l_L$$

$$j_{\alpha}^{\text{NC}} = \sum_{l=e,\mu,\tau} \bar{\nu}_{lL} \gamma_{\alpha} \nu_{lL}$$

II. Neutrino mass term

Neutrino nature is determined by the type of  
the neutrino mass term

Two completely different possibilities:

Majorana or Dirac?

### III. Neutrino mixing

$$\nu_{lL}(x) = \sum_i U_{li} \nu_{iL}(x)$$

$\nu_i$  are Majorana or Dirac

Neutrinos  $\nu_e$ ,  $\nu_\mu$  and  $\nu_\tau$  are produced and detected in mixed flavor states

$$a \rightarrow b + l^+ + \nu_i$$

Final neutrino state

$$|\nu_f\rangle = \sum_i |\nu_i\rangle \langle \nu_i, l^+ b | S | a \rangle$$

$|\nu_i\rangle$  is the state with momentum  $\vec{p}$  and

$$E_i \simeq p + \frac{m_i^2}{2p}.$$

Neutrino mass-squared differences are much smaller than  $E^2$

We have

$$\langle \nu_i l^+ b | S | a \rangle \simeq U_{li}^* \langle \nu_l l^+ b | S | a \rangle_{SM}$$

$\langle \nu_l l^+ b | S | a \rangle_{SM}$  is the SM matrix element of the process  $a \rightarrow b + l^+ + \nu_l$

$$|\nu_f\rangle = |\nu_l\rangle \langle \nu_l l^+ b | S | a \rangle_{SM}$$

Normalized states

$$|\nu_l\rangle = \sum_i U_{li}^* |\nu_i\rangle$$

In production and detection processes flavor lepton numbers are effectively conserved.

Matrix elements are given by the SM

In strong interaction quark flavor is conserved

$B_d$  and  $\bar{B}_d$  are produced in mixed states

$$|B_d\rangle = \frac{1}{2N} (|B_H\rangle + |B_L\rangle); \quad |\bar{B}_d\rangle = \frac{1}{2N} \frac{p}{q} (|B_H\rangle - |B_L\rangle)$$

Mixed states and translation invariance

Translations in space and time

$$x'_\alpha = x_\alpha + a_\alpha$$

If invariance under translations holds

$$|\Psi\rangle' = e^{i P a} |\Psi\rangle$$

$P_\alpha$  is the operator of the total momentum

$|\Psi\rangle$  and  $|\Psi\rangle'$  describe the *the same physical states*

In the case of mixed states

$$|\nu_l\rangle' = e^{i P a} |\nu_l\rangle = e^{-i \vec{p} \vec{a}} \sum_{l'} |\nu_{l'}\rangle \sum_i U_{l'i} e^{i E_i a^0} U_{li}^*$$

*Initial and transformed vectors describe different states*

There is no invariance under translation in time

*In transitions between different flavor neutrinos energy is not conserved (time-energy uncertainty relation)*

Evolution equation in QFT is Schrodinger equation

$$i \frac{\partial |\Psi(t)\rangle}{\partial t} = H |\Psi(t)\rangle$$

$$|\nu_l\rangle_t = \sum_i e^{-E_i t} U_{li}^* |\nu_i\rangle = \sum_{l'} |\nu_{l'}\rangle \sum_{i=1}^3 U_{l'i} e^{-iE_i t} U_{li}^*$$

Non stationary state

In the case of  $B_d$  and  $\bar{B}_d$

$$|B_d\rangle_t = g_+(t) |B_d\rangle + \frac{q}{p} g_-(t) |\bar{B}\rangle$$

$$|\bar{B}_d\rangle_t = g_-(t) \frac{p}{q} |B_d\rangle + g_+(t) |\bar{B}\rangle$$

$$g_{\pm}(t) = \frac{1}{2} (e^{-i\lambda_H t} \pm e^{-i\lambda_L t}); \lambda_{H,L} = m_{H,L} - \frac{i}{2} \Gamma_{H,L}$$

All successful phenomenology is based on these relations

Neutrino state is significantly changed at the time  $t$  which satisfies inequality

$$(\Delta E)_{ik} t = \frac{\Delta m_{ik}^2}{2E} t \geq 1$$

for at least one  $\Delta m_{ik}^2$

If we put  $t \simeq L$  (K2K, MINOS)

we come to the standard expression for  $\nu_l \rightarrow \nu_{l'}$  transition probability

$$P(\nu_l \rightarrow \nu_{l'}) = |\delta_{ll'} + \sum_{i>1} U_{l'i} (e^{-i \Delta m_{1i}^2 \frac{L}{2E}} - 1) U_{li}^*|^2$$

Time-energy uncertainty relation

$$\frac{\Delta m_{ik}^2}{2E} L \geq 1$$

is necessary condition to observe neutrino oscillations

Different assumptions exist on the physics which drive neutrino oscillations

We assumed that in

$$|\nu_l\rangle = \sum_i U_{li}^* |\nu_i\rangle$$

$|\nu_i\rangle$  are states with the same momentum

In approach based on Schrodinger evolution equation this is the only possibility compatible with experiment (otherwise enters arbitrary terms in oscillation phases)

Other assumption  $|\nu_i\rangle$  are states with the same energy

time in oscillations is not measured, only distance is relevant

oscillation probabilities are averaged over time  
( $E_i = E_k$ )

Contradict to K2K, MINOS

## Additional argument against equal energy assumption

Hamiltonian in matter

$$H = H_0 + H_I$$

$$(H_I)_{l'l} = \sqrt{2} G_F \rho_e \eta_{l'l}$$

The free Hamiltonian in the flavor representation if  $E_i = E$  is unit matrix

$$(H_0)_{l'l} = \langle \nu_{l'} | H_0 | \nu_l \rangle = \sum_i U_{l'i} E_i U_{li}^* = E \delta_{l'l}$$

**No MSW matter effect** observed in solar experiments

## Conclusion

1. Because neutrino mass-squared differences are very small **flavor neutrinos**  $\nu_e, \nu_\mu, \nu_\tau$ , produced and detected in weak processes, **are described by coherent mixed states**
2. **Neutrino oscillations and oscillations**  $B_d^0 \Leftrightarrow \bar{B}_d^0$  etc. **have the same origin.** The formalism which is based on Schrodinger evolution equation and on non stationary states was confirmed by numerous high-precision data.
3. **Time-energy uncertainty relation plays crucial role in oscillations.** Neutrino oscillations with final time during which neutrino state is significantly change **require uncertainty of energy.** This corresponds to violation of invariance under translation in time which takes place in the case of the mixed states.