Models of Neutrino Masses: A Brief Overview

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Why is neutrino mass physics important?

☞ **Two major unresolved puzzles in particle physics**

➢ (i) MASS PUZZLE

Why and How $< H > \neq 0$?

(Answer could come from searches for Higgs boson, SUSY, Extra Dimension etc.)

➢ FLAVOR PUZZLE

Why 3 generations ? Mixings among quarks and leptons; Origin of CP violation, strong CP etc.

(Answer could come from ν- mass physics, B-physics, Rare processes- $d^e_{n,e,\mu}$, $(\mu, \tau) \rightarrow (e, \mu) + \gamma$)

➢ Both vital to unravel the nature of new physics

$m_\nu \neq 0$ and Flavor Physics

☞

- $m_\nu = 0$ in the standard model; $\longrightarrow$ e, μ, τ are "mass locked" (no mixings) and therefore Leptons are " FLAVOR STERILE" !

- Once $m_\nu \neq 0$, leptons develop a full flavor physics.

- One may hope that in the true theory quark and lepton flavor physics may be related. (as in GUT theories) or it may reveal new symmetries.

Theoretical Challenges Posed by $m_\nu \neq 0$

☞

1. Why $m_\nu \ll m_{u,d,e}$?

2. Why are neutrino mixings so much largerthan quark mixings ?

3. How do neutrino fit into the big picture that relates to other particle physics issues ?

4. Nature of new physics depends crucially on whether neutrinos are Majorana or Dirac fermions

   (This talk assumes them to be Majorana type !!)

5. Are there new types of neutrinos (i.e. sterile neutrinos, ν_s)

Seesaw: The Dominant Paradigm:

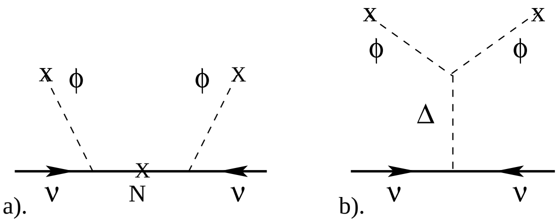

Figure 1: Seesaw diagram

- ☞ **Add ν_R to the standard model**

- ☞ $\rightarrow \mathcal{M}_\nu \simeq -\frac{h_\nu^2 v^2}{M_R}$.

This implies $m_{\nu_i} \ll m_{u,d,e...}$.

Seesaw mechanism

Implications of Seesaw

☞

- Neutrino is Majorana :Can lead to neutrinoless double beta decay and other $\Delta L = 2$ processes;
- Scale of the RH neutrino mass: roughly speaking:
  $M_{R,max} \simeq \frac{m_t^2}{\sqrt{\Delta m_A^2}} \simeq 10^{14} - 10^{15}$ GeV
  M_R close to the conventional SUSY GUT scale !!
- $M_R \ll M_{Pl}$: implies new symmetry of Nature: $B - L$
- N_R provides a simple way to understand the origin of matter via leptogenesis:

Testing seesaw with Lepton Flavor Violation

☞ **Large neutrino mixings imply large 23 and 33 elements of Dirac coupling $h_D\bar{L}HN_R$.**

Seesaw + supersymmetry →, superpartners remember high scale effects through radiative corrections inducing large 23 and 12 slepton mixing, which in turn can induce significant $\tau \to \mu + \gamma$ and $\mu \to e + \gamma$.

Typical Lepton flavor violation Predictions

☞ $B(\mu \to e + \gamma)$

Present upper limit:(Los Alamos MEGA expt: $B \leq 1.2 \times 10^{-11}$; MEG (PSI) goal: 10^{-14})

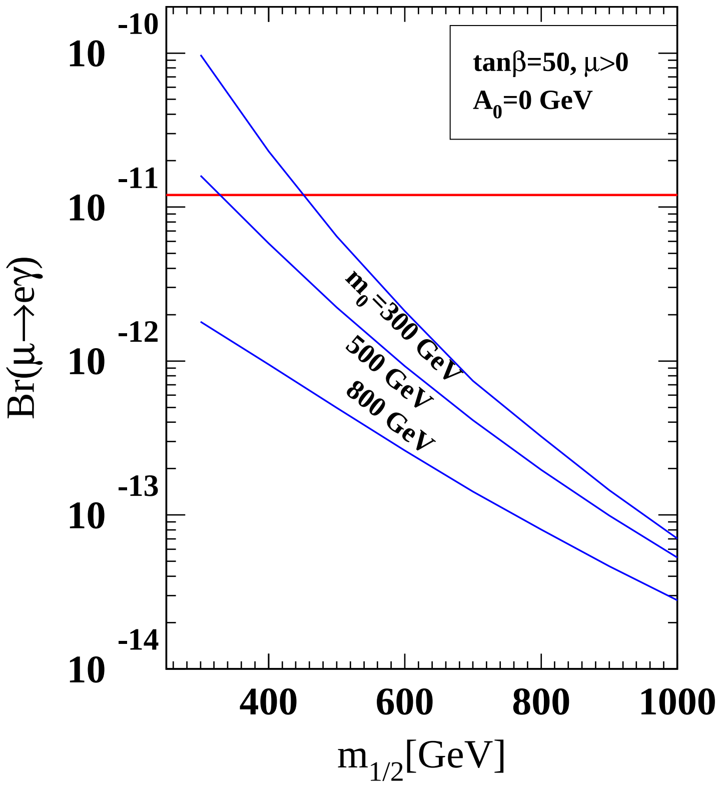

Figure 2: Typical predictions for $\mu \to e + \gamma$

$\tau \rightarrow (\mu, e) + \gamma$

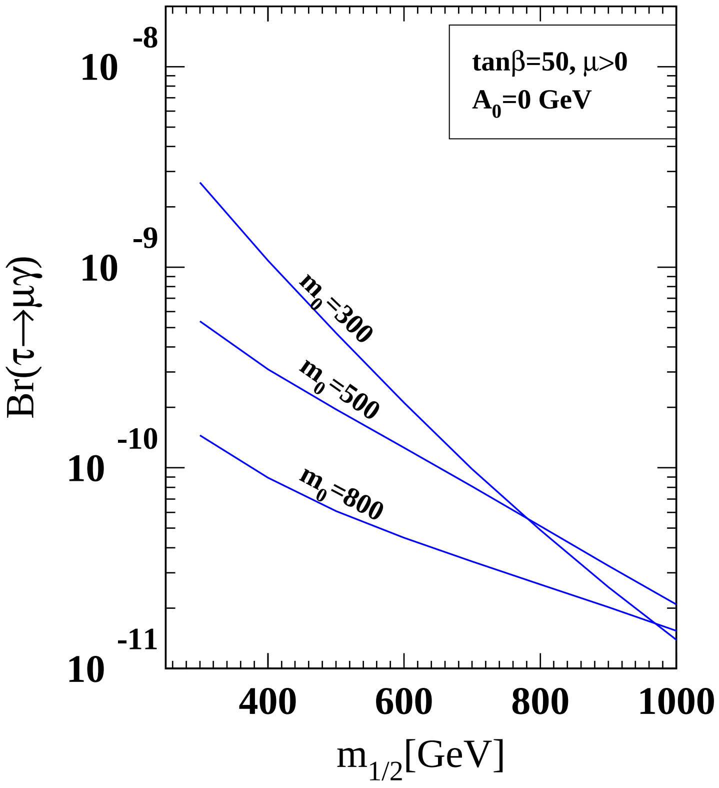

Figure 3: $\tau \longrightarrow \mu + \gamma$ in the same seesaw model

θ_{13} always nonzero in seesaw models

☞ **Radiative corrections (RGE from M_R down to M_Z) always induce nonzero θ_{13}**

rough minimum value

$$\sim \frac{\Delta m^2_{\odot}}{\Delta m^2_A} \times \frac{m^2_\tau tan^2\beta}{16\pi^2 v^2_W} \ln \frac{M_R}{M_Z} \simeq 0.001(tan\beta/50)^2$$

However if neutrino masses are degenerate, radiative corrections can magnify mixing angles.

How to Understand Large Mixings?

☞ **QUARK MIXINGS:** $\theta^q_{13} \simeq .23^\circ$; $\theta^q_{23} \simeq 2.3^\circ$ **and** $\theta^q_{12} \simeq 12.5^\circ$

Lepton mixings: SOLAR: $\theta_{12} \sim 34^o$

ATMOS: $\theta_{23} \simeq 45^o$

REACTOR: $\theta_{13} \leq 13^\circ$

Why so different- especially if quarks and leptons are to be unified at some scale ?

Quark-lepton unification and disparate mixing patterns

☞ **Theoretical ideas**

(i) New flavor structure for RH neutrinos:
$\mathcal{M}_\nu \simeq -M_D^T M_R^{-1} M_D$

e.g. Suppose $M_D \simeq M_q \sim \begin{pmatrix} m_1 & 0 \\ 0 & m_2 \end{pmatrix}$ **but**
$M_R \sim \begin{pmatrix} 0 & M \\ M & 0 \end{pmatrix}$

zero quark mixing but maximal neutrino mixing.

(ii) Type II seesaw (Triplet)

In asymptotically parity sym models,
$\mathcal{M}_\nu \simeq f\frac{v_{wk}^2}{v_R}$

$f \propto M_R$**; hence decouples from charged fermion masses.**

Radiative Corrections

Babu, Leung and Pantaleone; Chankowski and Pluciniek; Antusch, Kersten, Lindner and Ratz; Casas, Espinoza, Ibarra and Navarro

☞ **At high scale $\theta^q = \theta^l$- but if neutrinos are quasi-degenerate, it can magnify mixing angles:**

$$\frac{d\theta_{ij}}{dt} \sim \frac{h_\tau^2}{16\pi^2}\frac{m_i+m_j}{m_i-m_j} ln\frac{M_R}{M_Z}$$

Parida, Rajasekaran and RNM (03)

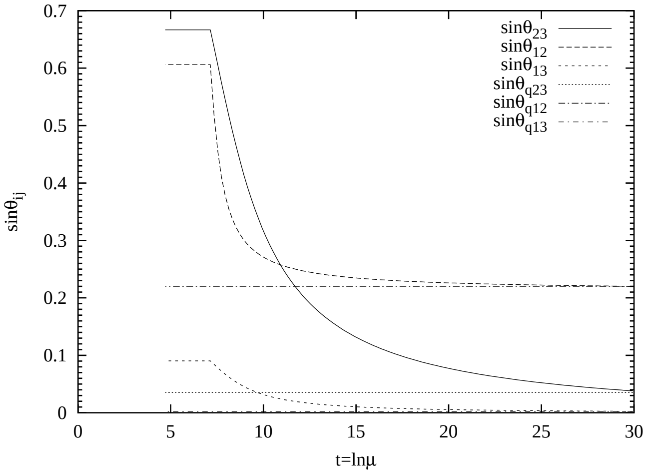

Figure 4: High scale mixing unification and Radiative magnification of mixing angles for degenerate neutrinos:

☞ **Requires $m_0 \geq 0.1$ eV; see Heidelberg-Moscow evidence. In any case testable in the next round $\beta\beta_{0\nu}$ search.**

MODEL BUILDING

☞ Two Approaches:

1. Bottom-Up:

Search for leptonic symmetries, mass matrix textures etc.

2. Top-Down: Grand unified theories:

Bottom-Up: Searching for symmetries of leptons

☞

- ➢ $\mu - \tau$ Symmetry inspired by maximal atmospheric and small θ_{13}
  mass matrix:
- ➢ $\mathcal{M}_\nu = \sqrt{\Delta m^2_A}\begin{pmatrix} d\epsilon^n & b\epsilon & a\epsilon \\ b\epsilon & 1+\epsilon & 1 \\ a\epsilon & 1 & 1+c\epsilon \end{pmatrix}; n \geq 1.$

  $a = b$ and $c = 1 \longrightarrow \theta_{23} = \pi/4$ and $\theta_{13} = 0$;
  The mass matrix has $\mu - \tau$ symmetry.
- ➢ Violations of these lead to $\theta_{13} \simeq \frac{\Delta m^2_\odot}{\Delta m^2_A} \simeq 0.04$ correlated to departure of θ_{23} from $\pi/4$: testable in planned experiments.

RNM (2004); Grimus, Lavoura, Joshipura, Kaneko, Tanimoto (2004); de Gouvea, Kitabayashi and Yasue; Lam, w/Rodejohann; w/Nasri, Yu

$\mu - \tau$ symmetry and θ_{13} and θ_{23} correlation.

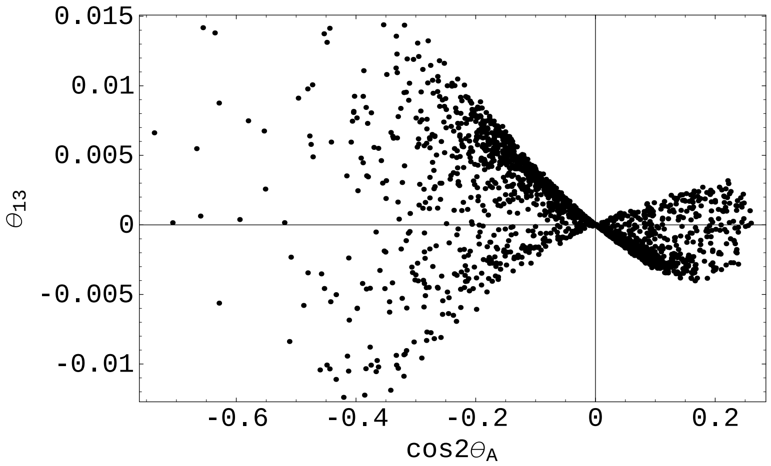

Figure 5: Departure from $\mu - \tau$ symmetry and correlation between θ_{13} and θ_A

Tri-bi-maximal mixing

Wolfenstein (78); Harrison, Perkins, Scott; Xing (2002)

☞ **Remarkable feature of neutrino mixing:**

$$U_{PMNS} = \begin{pmatrix} \frac{\sqrt{2}}{\sqrt{3}} & \frac{1}{\sqrt{3}} & 0 \\ -\frac{\sqrt{1}}{\sqrt{6}} & \frac{\sqrt{1}}{\sqrt{3}} & \frac{\sqrt{1}}{\sqrt{2}} \\ \frac{\sqrt{1}}{\sqrt{6}} & -\frac{\sqrt{1}}{\sqrt{3}} & \frac{\sqrt{1}}{\sqrt{2}} \end{pmatrix}$$

In a seesaw framework, this is compatible only with normal hierarchy !!

Rodejohann, Pfletinger (2005)

☞ **Is it possible that this particular value of the solar angle is an indication of a further underlying symmetry higher than the $\mu - \tau$ symmetry ?**

Many attempts to search for this symmetry:

☞ A_4 Ma; Babu and He; Altarelli and Ferruglio; Adhikary,Brahmachari, Ghosal, Ma, Parida;
Other symmetries King, Ross, Madeiros-Varzeilas; Caravaglios and Morrisi; Grimus, Lavoura..

An S_3 embedding of $\mu - \tau$ symmetry

☞ **A simple observation: tbm follows from a mass matrix which has the following structure (Nasri, Yu and RNM (06)):**

$M_\nu = M_0(S_3) + M_1(Z_2)$

where $M_o = \begin{pmatrix} a & b & b \\ b & a & b \\ b & b & a \end{pmatrix}$ **and the** $\mu - \tau$ **contribution**

$$M_1 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & c & -c \\ 0 & -c & c \end{pmatrix}$$

A simple way to get this is from a combination of type II and type I seesaw:

$M_\nu = f_T v_L - M_D^T M_R^{-1} M_D$ **where** $M_0 = f_T v_L$ **and** $M_D = (0, m, -m)$ **and only one right handed neutrino (or 3 RH nu's with** $M_{R,2,3} \gg M_{R,1}$ **)**

Solar angle and quark-lepton unification

☞ **Inspired by the observation that $\theta_\odot + \theta_C \simeq (46.7 \pm 2.4)^o \simeq \pi/4$; $\theta_A + V_{cb} + \pi/4$. Called Quark-Lepton complementarity**

Smirnov; Raidal; Minakata and Smirnov

☞ **Suppose tree level $\theta_{12} = 45^o$ and $\theta_C = 0$; since $U_\ell^\dagger U_\nu = U_{PMNS}$, then if $U_\ell = U_{CKM}$, then departure of solar mixing angle from 45^o is related to Cabibbo angle:**

$U_\ell = U_{CKM}$ is a signal of $SU(4)_c$.

Catch however is that: strict QLC requires that $U_{PMNS} = ...U^m_{23}...U^m_{12}U^{CKM\dagger}_{12}$- it is almost like a fine tuning condition;

CP violating phases must be appropriately placed etc.

Promise and Reality

☞ **Is it possible to have quark-lepton unified theory that leads to QLC?- perhaps approximately..**

Frampton and R. N. M.; Antusch, King and RNM (05)

☞ **Start with gauge theory based on $SU(2)_L \times SU(2)_R \times SU(4)_c$ and $U(1)_X$ where $X = F_1 - F_2 - F_3$ (behaves like $L_e - L_\mu - L_\tau$)**

Leading order: Inverted hierarchy and exact bimaximal mixing with $U_{CKM} \;=\; I$ in leading order;

Next order, $SU(4)_c$ gives $U_l \;=\; V_{CKM}$ leading to approximate QLC since matrices appear in the wrong order: $U_{PMNS} = U^{CKM\dagger} U^{bm}_\nu$.

Predicts $\theta_{13} \simeq 0.1$

m_ν and Grand unification

☞ **M_R close to M_U**

- raises the hope that seesaw scale and GUT scale have common origin

- Perhaps neutrino masses and mixings can be predicted due to higher symmetry of GUT theories

SO(10) SUSY GUT just right for neutrinos

☞

- ➢ unification of all **16** fermions of one generation
- ➢ $\begin{pmatrix} u & u & u & \nu \\ d & d & d & e \end{pmatrix}_{L,R}$ (a la Pati-Salam) fits into **16** dim. rep of SO(10);
- ➢ Contains the N_R needed for seesaw automatically
- ➢ Breaking SO(10) down
- ➢ Contains B-L symmetry whose breaking essential to get seesaw formula
- ➢ How one breaks B-L leads to different physics:

Breaking B-L

☞ Breaking B-L: 16 Higgs vrs 126 Higgs

- ➢ (a) **16**- Higgs breaking → no dark matter without additional assumptions
- ➢ (b)**126** Higgs breaking B-L leads automatically to dark matter: no additional symmetry needed
- ➢ Example of a theory with **126** breaking B-L

Minimal SUSY SO(10) For Neutrinos

Babu, RNM (92); Bajc, Senjanovic, Vissani (2002); Goh, RNM, Ng (03)

☞ What is minimal SO(10)

- ➢ $\mathbf{16}\otimes\mathbf{16}=\mathbf{10}\oplus\mathbf{126}\oplus 120$: choose one of each or less;
- ➢ Type II seesaw leads to sumrule:
  $f \;=\; 10^{-11}(M_d - M_\ell)$
  (Relation valid at GUT scale)
- ➢ At GUT scale $m_b \simeq m_\tau$

$$\mathcal{M}_\nu \;=\; m_b c\begin{pmatrix}\lambda^4 & \lambda^4 & \lambda^3\\ \lambda^4 & \lambda^2 & \lambda^2\\ \lambda^3 & \lambda^2 & \lambda^2\end{pmatrix} = m_b c\lambda^2\begin{pmatrix}\lambda^2 & \lambda^2 & \lambda\\ \lambda^2 & 1+\lambda & 1\\ \lambda & 1 & 1\end{pmatrix}$$

- Leads to large mixings θ_{23} and large θ_{12}; measurable θ_{13}
- $\sqrt{\frac{\Delta m^2_\odot}{\Delta m^2_A}} \sim \lambda$.

Lessons from the minimal SO(10)

☞ • There is an intimate relation between the quark and the lepton flavor pattern:

• Large maximal mixings need not necessarily imply symmetries.

☞ What about CP violation ?

• Two ways to go:

(i) minimal model with complex Yukawas:

Goh et al. (2003); Bertolini, Malinsky; Babu and Macesanu;(2005)..

☞ Six phases and connection between $b - \tau$ unification and large mixings is lost !!

CKM phase fit appears only in a small range of parameter range and requires larger m_s.

CP violation with 120 Higgs added:

Dutta, Mimura and RNM. (04,05)

☞ **Here the number of phases arise from the antisymmetric 120 coupling and there are only three more parameters.**

Large θ_A and $b - \tau$ mixing are still connected; the model is still predictive.

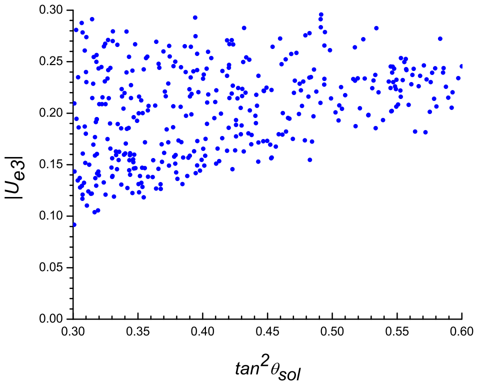

Figure 6: scatter corresponds to different allowed quark mass values

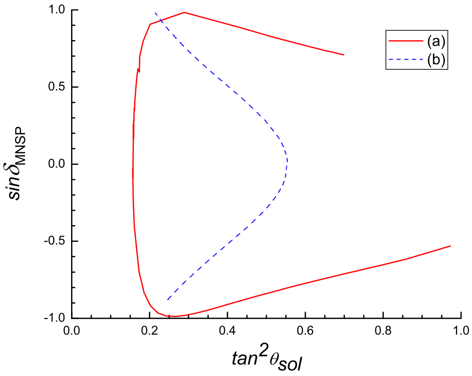

Figure 7: Prediction for the Dirac phase in SO(10) theory

SO(10) with 16-Higgs

Albright, Barr; modified version Ji, Li, RNM (05)

☞ **Apriori many couplings are allowed; to make model predictive, need to add additional flavor symmetries;**

Large mixings come from the charged lepton mixings Note $U_{PMNS} = U_l^\dagger U_\nu$.

Typically predicts large $B(\mu \rightarrow e + \gamma)$.

Main message of GUT approach to neutrinos

☞ **Relates quark flavor structure to lepton flavor structure:**

e.g. 126 models: $m_b \simeq m_\tau$ at M_U responsible for large neutrino mixings:

CKM parameter λ is the reason for m_2/m_3;

In 16 models, different connections

☞ **A key prediction from the quark-lepton linkage: Neutrino mass hierarchy is Normal !!**

What progress have we made towards a unified unravelling of quark and lepton flavor puzzle ?

☞ **We now have simple grand unified frameworks that correlate quark and lepton flavor structure making testable predictions !!**

Given their disparate flavor structure, it was not obvious that this could be done.

Next step would be to discover flavor symmetries that would make this flavor structure follow naturally and even perhaps predict as much of it as possible.

Possible such symmetries are S_4, A_4 or $SU(3)$ etc.

Some tests of the key theory ideas:

☞

Generic Seesaw	$\beta\beta_{0\nu}$
Generic SUSY Seesaw	$\beta\beta_{0\nu}$, $\mu,\tau\rightarrow e+\gamma$
Type II seesaw	**Quasi-deg neutrinos**
Leptogenesis	**CP phase at low energies**
Generic SO(10)	**Normal hierarchy**

☞ **Testing leptonic symmetries:**

$\mu-\tau$ **exchange sym.**	$\theta_{13}\leq 0.04$
QLC	$\theta_{13}\geq 0.08$
$\sim L_e-L_\mu-L_\tau$	**Inverted hierarchy (LBL,** $\beta\beta_{0\nu}$**)**

Other model building issues

☞ • If LSND is confirmed by Mini BooNe, we will need sterile neutrinos;

Other possible arguments for a sterile neutrino:

• Dark matter issues: CDM may be running into possible difficulty in explaining (a) flat central galactic density profile and (b) paucity of dwarf galaxies in local group;

KeV dark matter can come to the rescue Dodelson, Widrow; Abazajian, Fuller, Patel; ...

Evidence for a sterile neutrino will revolutionize the field again.

Issues with sterile neutrinos

☞ **Theoretical and Cosmological**

1. ν_s is singlet under standard model. Why its mass is not $M_{P\ell}$?
2. BBN allows at most 3.3 neutrinos; 3+1 or 3+2 scenarios for LSND → 4 or 5. (see however Cyburt et al (2004) for a more relaxed bound of 4.5). How to reconcile if BBN allows only $\delta N_\nu \leq 4$?
3. WMAP and SDSS limit $\Sigma_i m_i \leq 0.3 - 1$ eV or so (2σ) for ν's in equilibrium at BBN epoch or contributing to energy density at the recombination era.

BBN constraints

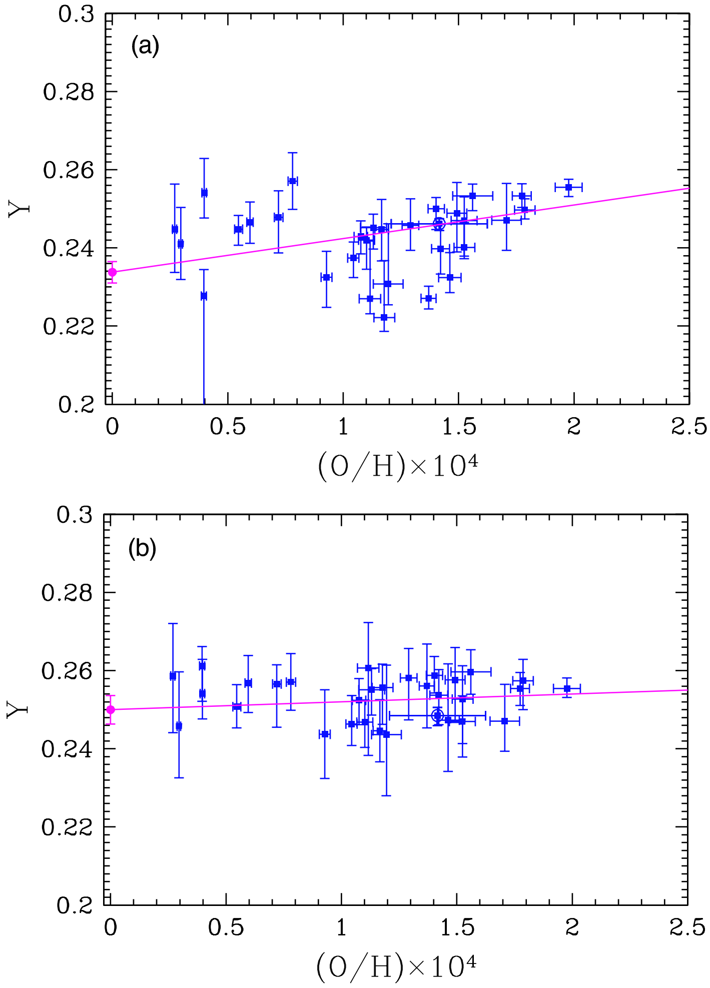

Figure 8: **Primordial Helium abundance determined from O/H from 30 HII regions and from NGC346A without and with stellar absorption corrections: $Y \sim 0.250 \pm 0.004$. The solid lines represent the linear fit. Fukugita and Kawasaki (06)**

BBN Constraints- contd.

☞

Observations	$\eta_{10} \equiv 10^{10}\eta$	N_ν	$\delta N_{\nu,max}$
Y_p + D/H$_A$	$5.94^{+0.56}_{-0.50}$	$3.14^{+0.70}_{-0.65}$	**1.59**
$Y_p + \eta_{CMB}$	6.14 ± 0.25	$3.08^{+0.74}_{-0.68}$	**1.63**
D/H$_A$ + η_{CMB}	6.16 ± 0.25	$3.59^{+1.14}_{-1.04}$	**2.78**
Y_p + D/H$_A$ + η_{CMB}	$6.10^{+0.24}_{-0.22}$	$3.24^{+0.61}_{-0.57}$	**1.44**

Constraints on KeV sterile neutrino WDM

☞ **Key question is how is keV ν_s produced after inflation ?**

(a) $\nu_{e,\mu,\tau} \longrightarrow \nu_s$
or
(b) some other way (see later)

If (a), there are strong constraints:

☞ **scenario either ruled out or on the verge of being.**

Seljak et al (2006);..

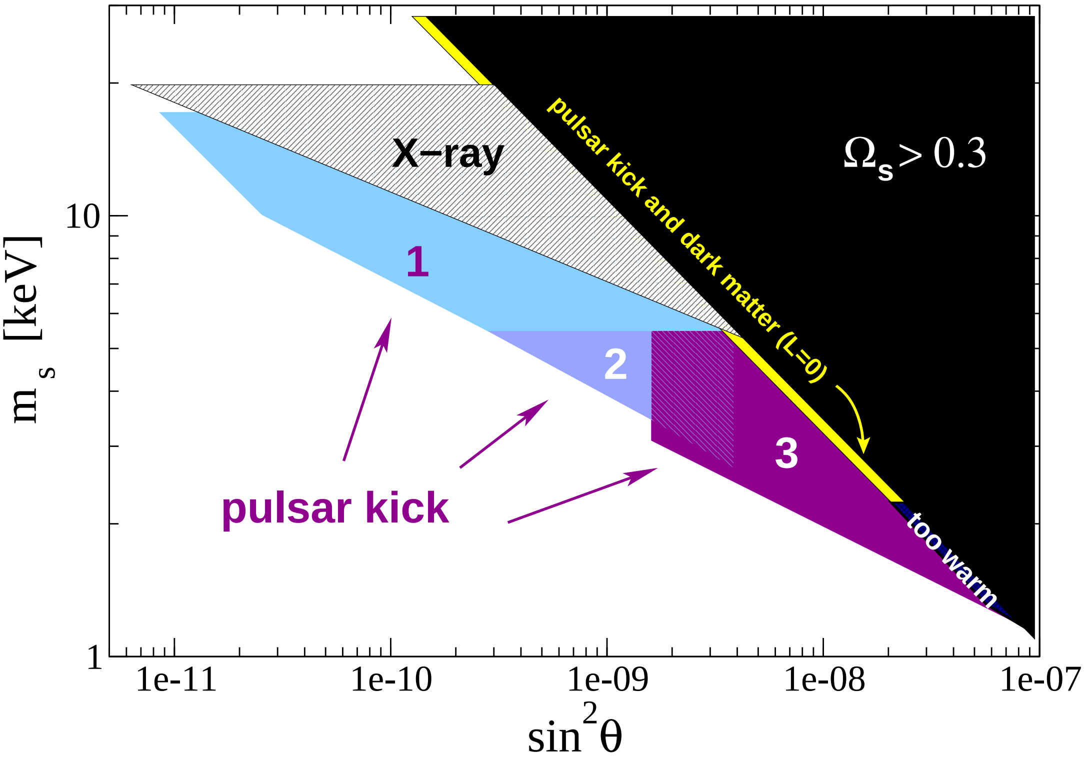

Figure 9: **Astrophysical and cosmological constraints on KeV neutrino dark matter**

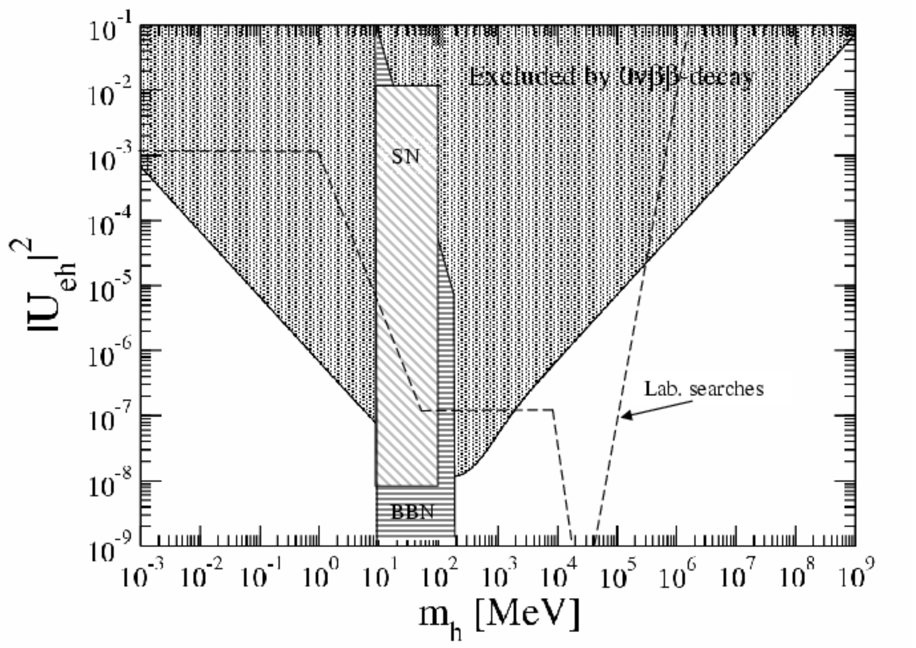

Figure 10: **Neutrinoless double beta decay and other constraints on sterile neutrinos**

Other constraints

Mirror models

☞ **Coexisting duplicate of the standard model-inspired by superstring theories**

➢ Standard model ⊗ Standard model-prime .

➢

visible sector $SU(2)_L \times U(1)_{I_{3R}}$ $\times U(1)_{B-L}$	mirror sector $SU(2)_L \times U(1)_{I_{3R}}$ $\times U(1)_{B-L}$
$W, Z, \gamma,$ gluons $\begin{pmatrix} u_L \\ d_L \end{pmatrix}$ u_R, d_R $\begin{pmatrix} \nu_L \\ e_L \end{pmatrix}$ e_R, N_R Higgs H, Δ_R	$W, Z, \gamma,$ gluons $\begin{pmatrix} u_L \\ d_L \end{pmatrix}$ u_R, d_R $\begin{pmatrix} \nu_L \\ e_L \end{pmatrix}$ e_R, N_R H', Δ'_R

Asymmetric inflation: an essential ingredient

☞ **inflation reheating temp is lower in the mirror sector compared to the visible one i.e.** $T'_R \ll T_R$ **.**

Consequence number density of mirror particles are down by a factor $(T'_R/T_R)^3$**. Thus** $T'_R \simeq T_R/10 \rightarrow \nu_s, \gamma'$ **etc do not mess up BBN**

(Kolb, Turner; Hodges, Berezhiani, Dolgov and R.N.M. (1995))

A keV WDM ν_s plus 3+2 scenario for LSND just fits into mirror models

☞

- Lightness of sterile neutrinos are connected to mirror B-L symmetry (e.g. mirror seesaw).
- Give effective operators: $(LH)^2/M; (L'H')/M'; (LH)(L'H')/M''$.
- $\mathcal{M}_{\nu,\nu_s} \simeq \frac{v_{wk}^2}{M}\begin{pmatrix} 1 & \beta\epsilon' \\ \beta\epsilon' & \beta^2\epsilon \end{pmatrix}$

  where $\beta = v_{wk}/v'_{wk}$; $\epsilon = M/M'$
- Possible to have mirror models for both keV and eV steriles.
- Scenario: keV ν_s primordial but density adjusted by asymmetric inflation to give Ω_{DM}.
- No need for active sterile mixing to have WDM and therefore no constraints from X-ray data.
- the other 2 ν_s could have eV range mass and explain LSND.

Conclusion

☞

- ➢ Model building- very much a work in progress ! though we have learnt several important things.
- ➢ Seesaw- a dominant paradigm with many interesting features: e.g. origin of matter, grand unification, low energy tests in LFV etc.
- ➢ Quark lepton symmetry and $B-L$ as a new symmetry of nature.
- ➢ SO(10) GUT is a natural group with many interesting models that will be tested by the next generation of planned neutrino expts;
- ➢ Bottom up scenarios do reveal many new symmetries which can throw light on the flavor structure of quarks. Bottom-Up and Top-Down approaches must meet in the middle.
- ➢ High precision searches for θ_{13}, mass ordering, CP phase, $\beta\beta_{0\nu}$ will "weed" out a lot of models

  More importantly, it will illuminate the origin of flavor structure for both quarks and leptons