

Duality in left-right symmetric seesaw

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Why are neutrinos so light ?

◇ A simple and elegant explanation – The seesaw mechanism

(Minkowski, 1977; Gell-Mann, Ramond & Slansky, 1979; Yanagida, 1979; Glashow, 1979; Mohapatra & Senjanović, 1980)

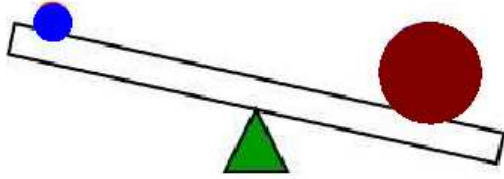
In addition:

Seesaw has a built-in mechanism for generating the baryon asymmetry of the Universe – Baryogenesis via Leptogenesis

(Fukugita & Yanagida, 1986; Luty, 1992; Covi et al., 1996; Buchmüller & Plümacher, 1996; ...)

The simplest version: Add 3 RH ν 's N_{Ri} to the minimal SM

Heavy N_{Ri} 's make ν_{Li} 's light :



$$-\mathcal{L}_{Y+m} = Y_\nu \bar{l}_L N_R H + \frac{1}{2} M_R N_R N_R + h.c.,$$

Neutrino mass matrix in the $(\nu_L, (N_R)^c)$ basis:

$$\mathcal{M}_\nu = \begin{pmatrix} 0 & m_D \\ m_D^T & M_R \end{pmatrix}$$

N_{Ri} are EW singlets $\Rightarrow M_R \sim M_{GUT}(M_I) \gg m_D \sim v$.

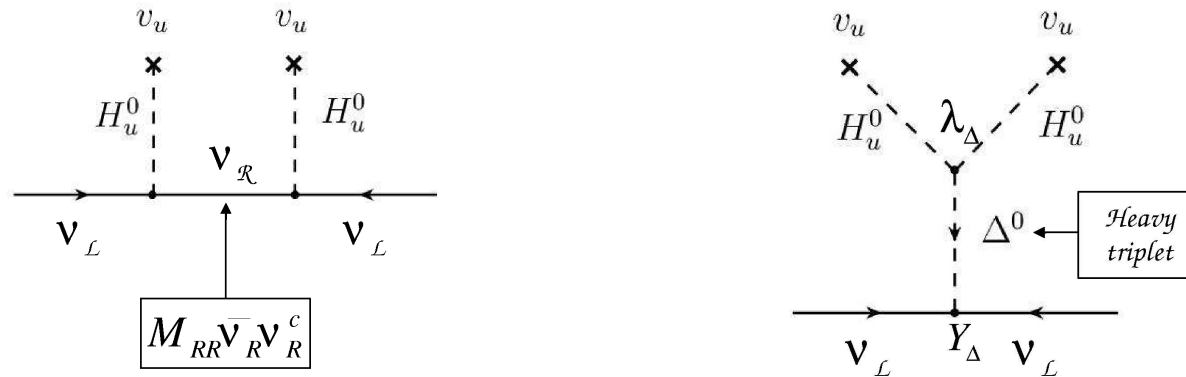
Block diagonalization: $M_N \simeq M_R$,

$$\diamond \quad \boxed{m_{\nu L} \simeq -m_D M_R^{-1} m_D^T} \quad \Rightarrow \quad m_\nu \sim \frac{(174 \text{ GeV})^2}{M_R}$$

For $m_\nu \lesssim 0.05 \text{ eV} \Rightarrow M_R \gtrsim 10^{15} \text{ GeV} \sim M_{GUT} \sim 10^{16} \text{ GeV} !$

Type II seesaw

Type I seesaw: Neutrino mass generated through exchange of a heavy RH ν 's N_R ; in type II – through exchange of a heavy $SU(2)_L$ - triplet scalars Δ_L :



In the SM, RH neutrinos N_R are singlets of the gauge group – “aliens”. They are more natural in Left-Right symmetric extensions of the SM: $SU(2)_L \times SU(2)_R \times U(1)_{B-L}$, $SU(2)_L \times SU(2)_R \times SU(4)_{PS}$, $SO(10)$, ...

LR symmetric models explain in a nice way maximal P- and C-violation in low- E weak interactions as a spontaneous symmetry breaking phenomenon – likely to be present in the final theory

In LR - symmetric models:

In general, both type I and type II seesaw contributions to m_ν are present :

$$\mathcal{M}_\nu = \begin{pmatrix} m_L & m_D \\ m_D^T & M_R \end{pmatrix}$$

Block diagonalization of \mathcal{M}_ν :

$$\diamond \quad m_\nu \simeq m_L - m_D M_R^{-1} m_D^T$$

m_D comes from Yukawa interactions with the bi-doublet Higgs Φ :

$$Y_1 \bar{l}_L \Phi l_R + Y_2 \bar{l}_L \tilde{\Phi} l_R, \quad \tilde{\Phi} \equiv \tau_2 \Phi^* \tau_2 \quad \Rightarrow \quad m_D = yv$$

m_L and M_R – from Yukawa couplings with triplet Higgses Δ_L and Δ_R

$$\Rightarrow \quad m_\nu \simeq f_L v_L - v^2 y (f_R v_R)^{-1} y^T$$

In general: m_ν , f_L and f_R – complex symmetric, y – complex matrix

Parity symmetry

Discrete LR symmetry (P) in LR-symm. models – two possible implementations:

$$(1) \quad l_{Li} \leftrightarrow l_{Ri}, \quad \Delta_L \leftrightarrow \Delta_R^*, \quad \Phi \leftrightarrow \Phi^\dagger$$

Yields

$$f_L = f_R^*, \quad y = y^\dagger,$$

$$(2) \quad l_{Li} \leftrightarrow l_{Li}^c \equiv (l_{Ri})^c, \quad \Delta_L \leftrightarrow \Delta_R, \quad \Phi \leftrightarrow \Phi^T$$

Yields

$$f_L = f_R \equiv f, \quad y = y^T.$$

Both implementations possible. Implem. (2) is more natural in $SO(10)$ GUTs (is an automatic gauge symmetry)

Adhere to the second implementation \Rightarrow

$$m_\nu = f v_L - v^2 y (v_R f)^{-1} y$$

For realization (1) of parity: type I term contains $(f^*)^{-1}$ instead of f^{-1} .

N_R , Δ_R , Δ_L are all at very high scale ($\sim 10^{12} - 10^{16}$ GeV) \Rightarrow no direct way of probing this sector of the theory.

◇ Neutrinos may provide a low-energy window into new physics at very high energy scales !

Bottom-up approach:

- Take m_ν from experiment
- Take y from data + theoretical assumptions (quark-lepton symm., GUTs)
- Solve the seesaw relation for f

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What would we like to know?

Try to learn as much as possible about the heavy neutrino sector

- What are the masses of heavy RH neutrinos?
- What are the mixing and CP violation in the heavy RH neutrino sector?
- Can all this give us a hint of the underlying theory?
- Is the solution that we found unique?
- Is type I or type II contribution in the light neutrino mass dominant or are they equally important?

Inverting the seesaw formula

Pure type I ($c_0 \equiv v^2/v_R$):

$$m_\nu = -c_0 (y f^{-1} y^T) \quad \Rightarrow \quad f = -c_0 (y^T m_\nu^{-1} y)$$

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General type I + II seesaw:

$$m_\nu = f v_L - \frac{v^2}{v_R} (y f^{-1} y)$$

– a nonlinear matrix equation for f . Yet can be readily solved analytically !

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Important point: A duality property of LR-symmetric seesaw



If f is a solution, so is $\hat{f} \equiv (m_\nu / v_L - f)$

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If f is a solution, so is $\hat{f} \equiv (m_\nu/v_L - f)$

\Rightarrow There is always an even number of solutions! ($\#sol. = 2^n$)

LR symmetry important for duality!

It was important that $f_L = f_R$ and $y^T = y$.

- ◇ For the other realization of parity symmetry ($f_L = f_R^*$ and $y = y^\dagger$) exactly the same duality holds !

Two-generation case

$$f^{-1} = \frac{1}{F} \begin{pmatrix} f_{22} & -f_{12} \\ -f_{12} & f_{11} \end{pmatrix}, \quad y = \begin{pmatrix} y_{e1} & y_{e2} \\ y_{\mu1} & y_{\mu2} \end{pmatrix},$$

$F \equiv \det f$, $y_{\mu1} = y_{e2}$. Define: $x \equiv v_L v_R / v^2$ and $m \equiv m_\nu / v_L \Rightarrow$

$$\begin{aligned} xF(f_{11} - m_{ee}) &= f_{22} y_{e1}^2 - 2f_{12} y_{e1} y_{e2} + f_{11} y_{e2}^2, \\ xF(f_{22} - m_{\mu\mu}) &= f_{22} y_{e2}^2 - 2f_{12} y_{e2} y_{\mu2} + f_{11} y_{\mu2}^2, \\ xF(f_{12} - m_{e\mu}) &= f_{22} y_{e1} y_{e2} - f_{12} (y_{e1} y_{\mu2} + y_{e2}^2) \\ &\quad + f_{11} y_{e2} y_{\mu2}, \end{aligned}$$

A system of coupled 3rd order equations

Admits a simple exact analytic solution !

Go to the basis where y is diagonal: $y = \text{diag}(y_1, y_2)$ (no loss of generality)

$$xF(f_{11} - m_{ee}) = f_{22} y_1^2$$

$$xF(f_{22} - m_{\mu\mu}) = f_{11} y_2^2$$

$$xF(f_{12} - m_{e\mu}) = -f_{12} y_1 y_2$$

Rescaling:

$$f_{ij} = \sqrt{\lambda} f'_{ij}, \quad m_{ij} = \sqrt{\lambda} m'_{ij}, \quad y_{ij} = \sqrt{\lambda} y'_{ij},$$

λ - arbitrary; fix it by requiring $F' \equiv \det f' = 1$. The system of eqs. for f'_{ij} becomes linear

Express f'_{ij} back through unprimed m_{ij} , $y_{1,2}$ and subst. into $F' = 1 \Rightarrow$
4th order characteristic equation for λ

The solution:

$$f = \frac{x\lambda}{(x\lambda)^2 - y_1^2 y_2^2} \begin{pmatrix} x\lambda m_{ee} + y_1^2 m_{\mu\mu} & m_{e\mu}(x\lambda - y_1 y_2) \\ \dots & x\lambda m_{\mu\mu} + y_2^2 m_{ee} \end{pmatrix}$$

λ has to be found from

$$\begin{aligned} [(x\lambda)^2 - y_1^2 y_2^2]^2 - x [\det m(x\lambda - y_1 y_2)^2 x\lambda \\ + (m_{ee} y_2 + m_{\mu\mu} y_1)^2 (x\lambda)^2] = 0 \end{aligned}$$

Has 4 complex solutions \Rightarrow 4 solutions for f which form 2 dual pairs

Determinant of the seesaw relation $x\hat{f} = -y f^{-1} y \Rightarrow x^2 F \hat{F} = y_1^2 y_2^2$

$F' = 1 \Rightarrow F = \lambda$; therefore

$$x^2 \lambda \hat{\lambda} = y_1^2 y_2^2$$

Allows to express the four solutions for λ in a simple closed form

Which seesaw type dominates ?

If $|m_{\alpha\beta}m_{\gamma\delta}| \gg 4|y_i y_j/x|$, one of the solutions λ_i satisfies $|x\lambda_1| \gg |y_i y_j|$

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Then for the dual solution λ_2 : $|x\lambda_2| \ll |y_i y_j|$

$\Rightarrow f_2 = \hat{f}_1$ corresponds to type I seesaw

The remaining 2 solutions are of mixed type.

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Similar situation in 3-generation case

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Using low-energy data only, it is impossible to say which seesaw type dominates (if any) – additional criteria necessary (leptogenesis?)

Three lepton generations

A system of six coupled 4th order equations – can be readily solved analytically by making use of the same trick as in 2-g case plus duality.

Characteristic equation for the rescaling factor λ is of 8th order \Rightarrow
8 solutions $\lambda_i \Rightarrow$ 8 solutions f_i ($i = 1, \dots, 8$).

Form 4 pairs of mutually dual solutions

For given y (i.e. m_D), all 8 solutions result in exactly the same mass matrix of light neutrinos m_ν !

Analysis of solutions under way

Conclusions

- In LR - symmetric models (minimal LR, Pati-Salam, $SO(10)$, E_6, \dots) one naturally has type I + II seesaw mechanism of neutrino mass generation
- Discrete LR symmetry (parity) leads to a relation between type I and type II contributions to m_ν , which results in a duality property of the seesaw formula: $f \iff m_\nu/v_L - f$
- For given y , there are 2^n (8 for three lepton generations) matrices f which result in exactly the same mass matrix of light neutrinos m_ν
- A simple analytic method developed for solving the seesaw nonlinear matrix equation. Allows bottom-up reconstruction of the Yukawa coupling matrix f of heavy RH neutrinos
- The results may be used for neutrino mass model building and studies of baryogenesis via leptogenesis. Allow to analytically explore the interplay of type I and II contributions to m_ν

Backup slides

RGE issues

The discrete LR (parity) symmetry of the underlying theory must be broken at some scale $v_{LR} \Rightarrow$ renormalization group evolution below this scale may result in a violation of conditions

$$f_L = f_R \equiv f, \quad y = y^T$$

at lower energies. Corrections to matrix elements of y and f depend logarithmically on ratios of masses of RH neutrinos and Higgs triplets; are suppressed by loop factors and possibly by small couplings. Numerically checked: If LR-violating corrections to the matr. elements are of the order of percent, reconstruction of the matrix f remains accurate at a percent level.

The stability may be lost if small matrix elements receive corrections proportional to the large elements \Rightarrow a dedicated study necessary

Mass matrix of light neutrinos m_ν

Symmetric mass matrix – diagonalized as

$$m_\nu = U^T m_{\text{diag}} U, \quad m_{\text{diag}} = \text{diag}(m_1, m_2, m_3)$$

The masses m_i known up to the overall scale parameterized by m_1 :

$$m_2 = \sqrt{m_1^2 + \Delta m_{21}^2}, \quad m_3 = \sqrt{m_1^2 + \Delta m_{31}^2}$$

Leptonic mixing matrix U :

$$U = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13}e^{-i\delta} \\ 0 & 1 & 0 \\ -s_{13}e^{i\delta} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & e^{i\alpha} & 0 \\ 0 & 0 & e^{i\beta} \end{pmatrix}$$

$$c_{ij} \equiv \cos \theta_{ij}, \quad s_{ij} \equiv \sin \theta_{ij}$$

Experiment: $\theta_{12} \simeq 34^\circ$, $\theta_{23} \simeq 45^\circ$, $\theta_{13} < 10^\circ$.

CP violating phases can be varied between 0 and 2π .

The solutions in 2-generation case

$$x\lambda_i = \frac{1}{4}[x \det m + r_{\pm} \pm \sqrt{2(\det m)^2 x^2 + 4kx + 2xr_{\pm} \det m}],$$

where

$$r_{\pm} = \pm \sqrt{(\det m)^2 x^2 + 4kx + 16y_1^2 y_2^2},$$

$$k = m_{ee}^2 y_2^2 + 2m_{e\mu}^2 y_1 y_2 + m_{\mu\mu}^2 y_1^2.$$

When λ_1 satisfies $|x\lambda_1| \gg |y_i y_j| \Rightarrow f_1 \simeq m$ (type II seesaw case)

The dual solution $x\lambda_2 = x\hat{\lambda}_1 = y_1^2 y_2^2 / (x\lambda_1)$ has modulus $\ll |y_i y_j| \Rightarrow f_2 = \hat{f}_1$ takes type I seesaw form

Condition $|x\lambda_1| \gg |y_i y_j|$ can only be satisfied when $|m_{\alpha\beta} m_{\gamma\delta}| \gg 4|y_i y_j / x|$.

This ensures the existence of solutions with one seesaw type dominance.

But: in general not all 4 solutions correspond to one seesaw type dominance in this limit: if $|\det m| \gg 4|y_i y_j / x|$, the remaining 2 dual solutions f_3 and f_4 are of mixed type. When $|m_{\alpha\beta} m_{\gamma\delta}| \lesssim 4|y_i y_j / x|$, all 4 sols. are of mixed type.

Three lepton generations

Once again go to the basis where $y = \text{diag}(y_1, y_2, y_3)$.

$$f^{-1} = \frac{1}{F} \begin{pmatrix} f_{22}f_{33} - f_{23}^2 & \dots & \dots \\ \dots & \dots & \dots \\ \dots & \dots & \dots \end{pmatrix}$$

System of 6 coupled 4th order equations:

$$x F (m_{11} - f_{11}) = y_1^2 (f_{22}f_{33} - f_{23}^2)$$

.....

Simple rescaling would not do! Use similar equations for duals:

$$x \hat{F} (m_{11} - \hat{f}_{11}) \equiv x \hat{F} f_{11} = y_1^2 (\hat{f}_{22}\hat{f}_{33} - \hat{f}_{23}^2)$$

and $(\hat{f}_{22}\hat{f}_{33} - \hat{f}_{23}^2) = (m_{22} - f_{22})(m_{33} - f_{33}) - (m_{23} - f_{23})^2$

Allows to express quadratic in f terms through linear and f -indep. terms and \hat{F}

Resulting system:

$$x (F + \hat{F}) f_{11} = y_1^2 [(m_{22} m_{33} - m_{23}^2) - (m_{22} f_{33} + m_{33} f_{22} - 2m_{23} f_{23})] + x F m_{11}$$

.....

Now one can rescale:

$$f_{ij} = \lambda^{1/3} f'_{ij}, \quad m_{ij} = \lambda^{1/3} m'_{ij}, \quad y_{ij} = \lambda^{1/3} y'_{ij},$$

Fix λ by requiring $F' \equiv \det f' = 1$

Determinant condition: $x^3 F \hat{F} = -y_1^2 y_2^2 y_3^2 \Rightarrow$

$$x^3 \hat{F}' = - (y'_1 y'_2 y'_3)^2$$

System of 6 linear eqs. for f'_{ij} – easily solved. Substitution into $F' = 1$ ($F = \lambda$) \Rightarrow 8th order characteristic eq. for λ . Yields 4 pairs of dual solutions for the matrix f .

3 generation case – more details

Equations for the matrix elements of f and its dual \hat{f} :

$$xF(f_{ij} - m_{ij}) = y_i y_j F_{ij} , \quad (1)$$

$$x\hat{F}(\hat{f}_{ij} - m_{ij}) = -x\hat{F}f_{ij} = y_i y_j \hat{F}_{ij} , \quad (2)$$

Here: $F \equiv \det f$, $\hat{F} \equiv \det \hat{f}$ and

$$F_{ij} \equiv \frac{1}{2} \epsilon_{ikl} \epsilon_{jmn} f_{km} f_{ln} ,$$

$$\hat{F}_{ij} \equiv \frac{1}{2} \epsilon_{ikl} \epsilon_{jmn} \hat{f}_{km} \hat{f}_{ln} = M_{ij} - T_{ij} + F_{ij} ,$$

$$M_{ij} \equiv \frac{1}{2} \epsilon_{ikl} \epsilon_{jmn} m_{km} m_{ln} , \quad T_{ij} \equiv \epsilon_{ikl} \epsilon_{jmn} f_{km} m_{ln} .$$

A system of 6 coupled quartic equations for f_{ij} . RH sides are quadratic rather than linear in $f_{ij} \Rightarrow$ a simple rescaling would not linearize the system.

But: using the dual system of eqs. gives for these RH sides

$$y_i y_j F_{ij} = -x\hat{F}f_{ij} + y_i y_j (T_{ij} - M_{ij})$$

Linearization: rescaling by a factor $\lambda^{1/3}$

Determinant condition: $x^3 F \hat{F} = -y_1^2 y_2^2 y_3^2$. Fix λ by requiring $F'(\lambda) = 1$;
 $\Rightarrow \hat{F}' = -(y_1' y_2' y_3')^2 / x^3$. The linearized system:

$$[x^3 - (y_1' y_2' y_3')^2] f'_{ij} - x^3 m'_{ij} = x^2 y_i' y_j' (T'_{ij} - M'_{ij}).$$

Simplified case: $y_1 \rightarrow 0$ (physically well motivated). The result:

$$\begin{aligned} f_{11} &= m_{ee}, & f_{12} &= m_{e\mu}, & f_{13} &= m_{e\tau}, \\ f_{23} &= \left(m_{\mu\tau} + \frac{y_2 y_3 m_{e\mu} m_{e\tau}}{x\lambda} \right) / d_2, \\ f_{22} &= \left[m_{\mu\mu} + \frac{y_2^2}{x\lambda} \left(M_{22} - \frac{y_3^2 m_{ee} m_{e\mu}^2}{x\lambda} \right) \right] / d_1, \\ f_{33} &= \left[m_{\tau\tau} + \frac{y_3^2}{x\lambda} \left(M_{33} - \frac{y_2^2 m_{ee} m_{e\tau}^2}{x\lambda} \right) \right] / d_1, \end{aligned}$$

Here:

$$d_1 = 1 - \frac{y_2^2 y_3^2 m_{ee}^2}{(x\lambda)^2}, \quad d_2 = 1 + \frac{y_2 y_3 m_{ee}}{x\lambda}.$$

Characteristic equation for λ :

$$\lambda^4 \left\{ [(x\lambda)^2 - m_{ee}^2 y_2^2 y_3^2]^2 - x [\det m(x\lambda - m_{ee} y_2 y_3)^2 x\lambda + (M_{22} y_2 + M_{33} y_3)^2 (x\lambda)^2] \right\} = 0.$$

If a solution λ of general charact. eq. $\neq 0$ in the limit $y_1 \rightarrow 0$, then $\hat{\lambda} = -y_1^2 y_2^2 y_3^2 / (x^3 \lambda) \rightarrow 0 \Rightarrow$ determinant of \hat{f} vanishes. \Rightarrow The dual of any solution that is finite for $y_1 \rightarrow 0$ becomes singular and must be discarded.

\Rightarrow For $y_1 \rightarrow 0$ there are only 4 (rather than 8) solutions with no duals. The corresp. values of λ are zeros of the factor in curly brackets (which is quartic in λ).

A strong connection with the pure 2-g case

- A **different duality** among the 4 remaining solutions:

If λ satisfies the charact. equation, so does

$$\tilde{\lambda} \equiv y_2^2 y_3^2 m_{ee}^2 / (x^2 \lambda),$$

and it corresponds to

$$\tilde{f} \equiv \tilde{m} - f,$$

where

$$\tilde{m}_{\alpha\beta} = m_{\alpha\beta} + m_{e\alpha} m_{e\beta} / m_{ee}.$$

There are two pairs of such solutions.

Why is $m_\nu = 0$ in the Standard Model ?

- No RH neutrinos N_{Ri} – Dirac mass terms cannot be introduced
- Operators of the kind $llHH$, which could produce Majorana neutrino mass after $H \rightarrow \langle H \rangle$, are dimension 5 and so cannot be present at the Lagrangian level in a renormalizable theory
- These operators cannot be induced in higher orders either (even nonperturbatively) because they would break not only lepton number L but also $B - L$, which is exactly conserved in the SM

In the Standard Model:

B and L are accidental symmetries at the Lagrangian level. Get broken at 1-loop level due the axial (triangle) anomaly. But: their difference $B - L$ is still conserved and is an exact symmetry of the model