Övning #9

Tvåpartikelsystem

$$\psi(\bar{r}_1, \bar{r}_2, t)$$

Bestäms alltid av S.E.

$$i\hbar \frac{\partial \psi}{\partial t} = H\psi$$

där

$$H = -\frac{\hbar^2}{2m}\nabla_1^2 - \frac{\hbar^2}{2m}\nabla_2^2 + V(\bar{r}_1, \bar{r}_2, t)$$

Bosoner och fermioner:

$$\psi_\pm(\bar{r}_1, \bar{r}_2) = A[\psi_a(\bar{r}_1)\psi_b(\bar{r}_2) \pm \psi_b(\bar{r}_1)\psi_a(\bar{r}_2)]$$

Normering ger $A = \frac{1}{\sqrt{2}}$.

I allmänhet:

$$\langle (x_1 - x_2)^2 \rangle = \langle x_1^2 \rangle + \langle x_2^2 \rangle - 2\langle x_1 x_2 \rangle$$

För särskiljbara partiklar $\langle x_1 x_2 \rangle = \langle x \rangle_a \langle x \rangle_b$

För identiska partiklar:

$$\langle x_1 x_2 \rangle = \langle x \rangle_a \langle x \rangle_b \pm |\langle x \rangle_{ab}|^2$$

där $\langle x \rangle_{ab} = \int x \psi_a(x) \psi_b(x) dx$

4.38 Tre-dimensionell harmonisk oscillator

$$V(r) = \frac{1}{2}m\omega^2 r^2$$

a) I kartesiska koordinater

$$V(r) = \frac{1}{2}m\omega^2(x^2+y^2+z^2)$$

Således

$$E\psi(x,y,z) = -\frac{\hbar^2}{2m}\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}+\frac{\partial^2}{\partial z^2}\right)\psi + \frac{1}{2}m\omega^2(x^2+y^2+z^2)\psi$$

Potentialen oberoende i x, y, z-riktning $\Rightarrow \psi(x,y,z)$ oberoende

$$\psi(x,y,z) = X(x)Y(y)Z(z)$$

Stoppa in detta i S.E och dividera med XYZ

$$E = -\frac{\hbar^2}{2m}\frac{1}{X}\frac{\partial^2 X}{\partial x^2} + \frac{1}{2}m\omega^2x^2 - \frac{\hbar^2}{2m}\frac{1}{Y}\frac{\partial^2 Y}{\partial y^2} + \frac{1}{2}m\omega^2 y^2 - \frac{\hbar^2}{2m}\frac{1}{Z}\frac{\partial^2 Z}{\partial z^2} + \frac{1}{2}m\omega^2 z^2$$

Vänsterledet är konstant och vi har 3 delar enbart beroende av x, y, z respektive. Dessa måste vara konstanta. Således

$$-\frac{\hbar^2}{2m}\frac{1}{X}\frac{\partial^2 X}{\partial x^2} + \frac{1}{2}m\omega^2x^2 = E_x$$

$$-\frac{\hbar^2}{2m}\frac{1}{Y}\frac{\partial^2 Y}{\partial y^2} + \frac{1}{2}m\omega^2y^2 = E_y$$

$$-\frac{\hbar^2}{2m}\frac{1}{Z}\frac{\partial^2 Z}{\partial z^2} + \frac{1}{2}m\omega^2z^2 = E_z$$

3 harmoniska oscillatorer med

$$E_x = (n_x+\tfrac{1}{2})\hbar\omega, \quad E_y = (n_y+\tfrac{1}{2})\hbar\omega, \quad E_z = (n_z+\tfrac{1}{2})\hbar\omega$$

och

$$E = E_x+E_y+E_z = (n_x+n_y+n_z+\tfrac{3}{2})\hbar\omega = (n+\tfrac{3}{2})\hbar\omega, \quad n = 0,1,2,\ldots$$

b) Degenereringen av E_n, dvs på hur många sätt kan vi få energin E_n.

Om $n_x = n$, $n_y = n_z = 0$ ett sätt

$n_x = n-1$, $(n_y=1, n_z=0)$, $(0,1)$ två

$n_x = n-2$, $(n_y=2, n_z=0)$, $(1,1)$, $(0,2)$ tre o.s.v

Antalet sätt blir summan av alla dessa fall

$$d(n) = 1+2+3+\ldots+(n+1) = \frac{(n+1)(n+2)}{2}$$

4.55 Elektron i väteatom upptar tillståndet

$$R_{21}\left(\sqrt{\tfrac{1}{3}}\,Y_1^0\chi_+ + \sqrt{\tfrac{2}{3}}\,Y_1^1\chi_-\right) = \psi_{210}\cdot\sqrt{\tfrac{1}{3}}\,\chi_+ + \psi_{211}\sqrt{\tfrac{2}{3}}\,\chi_-$$

a) Mätning av L^2. För det här tillståndet $l=1$

$$\Rightarrow L^2 = l(l+1)\hbar^2 = 2\hbar^2 \quad \text{(alltid, dvs } P=1)$$

b) $L_z = 0, \hbar$

$P_0 = \frac{1}{3}$, $P_1 = \frac{2}{3}$

c) S^2, alltid $\frac{3\hbar^2}{4} \leftarrow s(s+1)\hbar^2$ för spinn-½ partikel

d) $S_z = \pm\frac{\hbar}{2}$

$P_+ = \frac{1}{3}$, $P_- = \frac{2}{3}$

e) $\bar{J} = \bar{L} + \bar{S}$

Utveckla basen $|l,m\rangle|s\, m_s\rangle$ i $|j, m_j\rangle$. Använd C-G

$$\sqrt{\tfrac{1}{3}}\,|1\,0\rangle|\tfrac{1}{2}\,\tfrac{1}{2}\rangle + \sqrt{\tfrac{2}{3}}\,|1\,1\rangle|\tfrac{1}{2}\,-\tfrac{1}{2}\rangle =$$

$$= \frac{1}{\sqrt{3}}\left(\sqrt{\tfrac{2}{3}}\,|\tfrac{3}{2}\,\tfrac{1}{2}\rangle - \sqrt{\tfrac{1}{3}}\,|\tfrac{1}{2}\,\tfrac{1}{2}\rangle\right) + \sqrt{\tfrac{2}{3}}\left(\sqrt{\tfrac{1}{3}}\,|\tfrac{3}{2}\,\tfrac{1}{2}\rangle + \sqrt{\tfrac{2}{3}}\,|\tfrac{1}{2}\,\tfrac{1}{2}\rangle\right)$$

$$= \left(\frac{\sqrt{2}}{3} + \frac{\sqrt{2}}{3}\right)|\tfrac{3}{2}\,\tfrac{1}{2}\rangle + \left(\frac{-1}{3} + \frac{2}{3}\right)|\tfrac{1}{2}\,\tfrac{1}{2}\rangle = \frac{2\sqrt{2}}{3}\,|\tfrac{3}{2}\,\tfrac{1}{2}\rangle + \frac{1}{3}\,|\tfrac{1}{2}\,\tfrac{1}{2}\rangle$$

$j = \frac{1}{2}, \frac{3}{2}$, $m_j = \frac{1}{2}$

$J^2 = \frac{3}{4}\hbar^2 \; (P = \frac{1}{9})$, $J^2 = \frac{15}{4}\hbar^2 \; (P = \frac{8}{9})$

f) $m_j = \frac{1}{2} \Rightarrow J_z = \frac{\hbar}{2}$, $P = 1$

g) $$|\psi|^2 = |R_{21}|^2 \left|\sqrt{\tfrac{1}{3}}\,Y_1^0\chi_+ + \sqrt{\tfrac{2}{3}}\,Y_1^1\chi_-\right|^2 = |R_{21}|^2\Big[\tfrac{1}{3}|Y_1^0|^2 \overbrace{\chi_+^\dagger\chi_+}^{=1}$$

$$+ \tfrac{2}{3}|Y_1^1|^2\underbrace{\chi_-^\dagger\chi_-}_{=1} + \tfrac{\sqrt{2}}{3}\,Y_1^0 Y_1^1\underbrace{\left(\chi_+^\dagger\chi_- + \chi_-^\dagger\chi_+\right)}_{=0}\Big] = |R_{21}|^2\left(\tfrac{1}{3}|Y_1^0|^2 + \tfrac{2}{3}|Y_1^1|^2\right)$$

$$= \{\text{från tabell}\} = \frac{1}{3}\cdot\frac{1}{24}\cdot\frac{1}{a^3}\cdot\frac{r^2}{a^2}\,e^{-r/a}\left[\frac{3}{4\pi}\cos^2\theta + 2\cdot\frac{3}{8\pi}\sin^2\theta\right] =$$

$$= \frac{1}{96\pi}\cdot\frac{1}{a^3}\cdot\frac{r^2}{a^2}\,e^{-r/a}$$

h) $\frac{1}{3}|R_{21}|^2 \int \underbrace{|Y_1^0|^2}_{\text{spin-up coeff.}} \sin^2\Theta \, d\Theta \, d\phi = \frac{1}{3}|R_{21}|^2 = \frac{1}{3}\frac{1}{24a^3} r^2 e^{-r/a} = \frac{1}{72a^3} r e^{-r/a}$

5.5 a) Skriv ned Hamiltonianen för två icke-interagerande identiska partiklar i den oändliga potentialbrunnen.

$$-\frac{\hbar^2}{2m}\frac{\partial^2\psi}{\partial x_1^2} - \frac{\hbar^2}{2m}\frac{\partial^2\psi}{\partial x_2^2} = E\psi \qquad 0\le x_1,x_2\le a \text{ , annars } \psi=0$$

$$\psi(x_1,x_2) = \frac{\sqrt{2}}{a}\left[\sin\left(\frac{\pi x_1}{a}\right)\sin\left(\frac{2\pi x_2}{a}\right) - \sin\left(\frac{2\pi x_1}{a}\right)\sin\left(\frac{\pi x_2}{a}\right)\right]$$

Bestäm E

$$\frac{d^2\psi}{dx_1^2} = -\left(\frac{\pi}{a}\right)^2\sin\left(\frac{\pi x_1}{a}\right)\sin\left(\frac{2\pi x_2}{a}\right) + \left(\frac{2\pi}{a}\right)^2\sin\left(\frac{2\pi x_1}{a}\right)\sin\left(\frac{\pi x_2}{a}\right)$$

$$\frac{d^2\psi}{dx_2^2} = -\left(\frac{2\pi}{a}\right)^2\sin\left(\frac{\pi x_1}{a}\right)\sin\left(\frac{2\pi x_2}{a}\right) + \left(\frac{\pi}{a}\right)^2\sin\left(\frac{2\pi x_1}{a}\right)\sin\left(\frac{\pi x_2}{a}\right)$$

$$\Rightarrow E\psi = -\frac{\hbar^2}{2m}\left(-\left(\frac{\pi}{a}\right)^2-\left(\frac{2\pi}{a}\right)^2\right)\sin\left(\frac{\pi x_1}{a}\right)\sin\left(\frac{2\pi x_2}{a}\right)$$

$$-\frac{\hbar^2}{2m}\left(\left(\frac{2\pi}{a}\right)^2+\left(\frac{\pi}{a}\right)^2\right)\sin\left(\frac{2\pi x_1}{a}\right)\sin\left(\frac{\pi x_2}{a}\right) =$$

$$= \frac{5\pi^2\hbar^2}{2ma^2}\psi$$

$$\Rightarrow E = \frac{5\pi^2\hbar^2}{2ma^2}$$

b) Hitta nästa två exiterade tillstånd om särskiljbara, identiska bosoner, identiska fermioner

Särskiljbara partiklar:

$$\psi_{22} = \left(\frac{2}{a}\right)\sin\left(\frac{2\pi x_1}{a}\right)\sin\left(\frac{2\pi x_2}{a}\right) \qquad \text{(icke-degenererad)}$$

$$\begin{cases} \psi_{13} = \left(\frac{2}{a}\right)\sin\left(\frac{\pi x_1}{a}\right)\sin\left(\frac{3\pi x_3}{a}\right) \\ \psi_{31} = \left(\frac{2}{a}\right)\sin\left(\frac{3\pi x_1}{a}\right)\sin\left(\frac{\pi x_3}{a}\right) \end{cases} \qquad \text{tvåfaldigt degenererad}$$

Identiska bosoner:

$$\psi_{22} = \left(\frac{2}{a}\right)\sin\left(\frac{2\pi x_1}{a}\right)\sin\left(\frac{2\pi x_2}{a}\right)$$

$$\psi_{13} = \frac{\sqrt{2}}{a}\left(\sin\left(\frac{\pi x_1}{a}\right)\sin\left(\frac{3\pi x_2}{a}\right) + \sin\left(\frac{3\pi x_1}{a}\right)\sin\left(\frac{\pi x_2}{a}\right)\right]$$

Identiska fermioner:

$$\psi_{13} = \frac{\sqrt{2}}{a}\left[\sin\left(\frac{\pi x_1}{a}\right)\sin\left(\frac{3\pi x_2}{a}\right) - \sin\left(\frac{3\pi x_1}{a}\right)\sin\left(\frac{\pi x_2}{a}\right)\right]$$

$$\psi_{23} = \frac{\sqrt{2}}{a}\left[\sin\left(\frac{2\pi x_1}{a}\right)\sin\left(\frac{3\pi x_2}{a}\right) - \sin\left(\frac{3\pi x_1}{a}\right)\sin\left(\frac{2\pi x_2}{a}\right)\right]$$

5.6 Två icke-interagerande partiklar med massan m. En är i tillståndet

$$\psi_n = \sqrt{\frac{2}{a}} \sin\left(\frac{n\pi x}{a}\right) \quad \text{(oändlig potentialbrunn)}$$

den andra i tillståndet ψ_ℓ. Bestäm $\langle (x_1 - x_2)^2 \rangle$.

a) Antag särskiljbara partiklar

$$(5.19) \quad \langle (x_1-x_2)^2\rangle_d = \langle x^2\rangle_a + \langle x^2\rangle_b - 2\langle x\rangle_a \langle x\rangle_b$$

De här separata väntevärdena har vi bestämt förut.

$$\langle x^2\rangle = a^2\left(\frac{1}{3} - \frac{1}{2(n\pi)^2}\right)$$

$$\langle x\rangle = \frac{a}{2}$$

$$\Rightarrow \langle (x_1-x_2)^2\rangle = a^2\left(\frac{1}{3} - \frac{1}{2(n\pi)^2}\right) + a^2\left(\frac{1}{3} - \frac{1}{2(m\pi)^2}\right) - 2\cdot\frac{a}{2}\cdot\frac{a}{2}$$

$$= 2\frac{a^2}{3} - \frac{a^2}{2} - a^2\frac{m^2+n^2}{2m^2n^2\pi^2} = \frac{a^2}{6} - a^2\frac{m^2+n^2}{2m^2n^2\pi^2} = a^2\left[\frac{1}{6} - \frac{1}{2\pi^2}\left(\frac{1}{n^2}+\frac{1}{m^2}\right)\right]$$

b) Antag identiska bosoner

Här $\langle (x_1-x_2)^2\rangle = \langle x_1^2\rangle + \langle x_2^2\rangle - 2\langle x_1 x_2\rangle$

$\langle x_1^2\rangle$ och $\langle x_2^2\rangle$ som förut $\langle x_1 x_2\rangle$ måste bestämmas.

I fallet av identiska bosoner har vi
fermioner

$$\langle x_1 x_2\rangle = \langle x\rangle_m \langle x\rangle_n \pm |\langle x\rangle_{mn}|^2$$

Vi känner $\langle x\rangle_m = \frac{a}{2}$. Kvar att bestämma är nu

$$\langle x\rangle_{mn} = \frac{2}{a}\int_0^a x \sin\left(\frac{n\pi x}{a}\right)\sin\left(\frac{m\pi x}{a}\right)dx = \frac{1}{a}\int_0^a x\left[\cos\left(\frac{(m-n)\pi x}{a}\right) - \cos\left(\frac{(m+n)\pi x}{a}\right)\right]dx$$

$$= \frac{1}{a}\left[x\frac{a}{(m-n)\pi}\sin\left(\frac{(m-n)\pi x}{a}\right) - x\cdot\frac{a}{(m+n)\pi}\sin\left(\frac{(m+n)\pi x}{a}\right)\right]_0^a - \int_0^a \frac{1}{(m-n)\pi}\sin\left(\frac{(m-n)\pi x}{a}\right) - \frac{1}{(m+n)\pi}\sin\left(\frac{(m+n)\pi x}{a}\right)$$

$$= +\left[\frac{a}{(m-n)^2\pi^2}\cos\left(\frac{(m-n)\pi x}{a}\right) - \frac{a}{(m+n)^2\pi^2}\cos\left(\frac{(m+n)\pi x}{a}\right)\right]_0^a =$$

$$= \frac{a}{\pi^2}\left[\frac{1}{(m-n)^2}\left((-1)^{(m-n)} - 1\right) - \frac{1}{(m+n)^2}\left[(-1)^{m+n} - 1\right]\right] =$$

$$= \frac{a}{\pi^2}\left((-1)^{m+n} - 1\right)\cdot\left(\frac{1}{(m-n)^2} - \frac{1}{(m+n)^2}\right) = \begin{cases} -\frac{a\cdot 8mn}{\pi^2(m^2-n^2)^2} & m,n \text{ motsatt paritet} \\ 0 & \text{annars} \end{cases}$$

Således:

$$\langle (x_1 - x_2)^2 \rangle = a^2\left[\frac{1}{6} - \frac{1}{2\pi^2}\left(\frac{1}{n^2} + \frac{1}{m^2}\right)\right] - \frac{128a^2m^2n^2}{\pi^4(m^2-n^2)^4}$$

c) Identiska fermioner => andra tecknet

$$\langle (x_1 - x_2)^2 \rangle = a^2\left[\frac{1}{6} - \frac{1}{2\pi^2}\left(\frac{1}{n^2} + \frac{1}{m^2}\right)\right] + \frac{128a^2m^2n^2}{\pi^4(m^2-n^2)^4}$$

5.7 Tre partiklar. En i tillstånd $\psi_a(x)$, en i $\psi_b(x)$ och en i $\psi_c(x)$.

Antag ψ_a, ψ_b, ψ_c ortonormala. Konstruera tre-partikeltillstånd för särskiljbara partiklar, identiska bosoner, identiska fermioner

a) Särskiljbara partiklar

$$\psi(x_1, x_2, x_3) = \psi_a(x_1)\psi_b(x_2)\psi_c(x_3)$$

b) Identiska bosoner. Tillståndet måste vara helt symmetriskt m.a.p byte av vilka två partiklar som helst.

$$\psi(x_1,x_2,x_3) = \frac{1}{\sqrt{6}}\Big[\psi_a(x_1)\psi_b(x_2)\psi_c(x_3) + \psi_a(x_1)\psi_c(x_2)\psi_b(x_3) + \psi_c(x_1)\psi_b(x_2)\psi_a(x_3)$$

$$+ \psi_b(x_1)\psi_a(x_2)\psi_c(x_3) + \psi_c(x_1)\psi_a(x_2)\psi_b(x_3) + \psi_b(x_1)\psi_c(x_2)\psi_a(x_3)\Big]$$

c) Identiska fermioner. Tillståndet måste vara helt anti-symmetriskt

$$\psi(x_1,x_2,x_3) = \frac{1}{\sqrt{6}}\Big[\psi_a(x_1)\psi_b(x_2)\psi_c(x_3) - \psi_a(x_1)\psi_c(x_2)\psi_b(x_3)$$

$$- \psi_c(x_1)\psi_b(x_2)\psi_a(x_3) - \psi_b(x_1)\psi_a(x_2)\psi_c(x_3) + \psi_c(x_1)\psi_a(x_2)\psi_b(x_3)$$

$$+ \psi_b(x_1)\psi_c(x_2)\psi_a(x_3)\Big]$$

5.11 a) Bestäm $\langle 1/|\bar{r}_1-\bar{r}_2|\rangle$ för tillståndet

$$\psi_0(\bar{r}_1,\bar{r}_2)=\frac{8}{\pi a^3}e^{-2(r_1+r_2)/a}$$

$$\left\langle\frac{1}{|\bar{r}_1-\bar{r}_2|}\right\rangle=\iint\frac{1}{|\bar{r}_1-\bar{r}_2|}|\psi_0(\bar{r}_1,\bar{r}_2)|^2\,d\bar{r}_1 d\bar{r}_2=\left(\frac{8}{\pi a^3}\right)^2\iint\frac{e^{-4(r_1+r_2)/a}}{\sqrt{r_1^2+r_2^2-2r_1r_2\cos\theta}}\,d\bar{r}_2\,d\bar{r}_1$$

Börja med integralen över $d\bar{r}_2$

$$\int\frac{e^{-4(r_1+r_2)/a}}{\sqrt{r_1^2+r_2^2-2r_1r_2\cos\theta_2}}r_2^2\sin\theta_2\,dr_2\,d\theta_2\,d\phi_2=2\pi\int r_2^2e^{-4(r_1+r_2)/a}dr_2\int\frac{\sin\theta_2}{\sqrt{r_1^2+r_2^2-2r_1r_2\cos\theta_2}}d\theta_2$$

$$=2\pi\int r_2^2e^{-4(r_1+r_2)/a}dr_2\left[+\frac{1}{r_1r_2}\sqrt{r_1^2+r_2^2-2r_1r_2\cos\theta}\right]_0^\pi=2\pi\int\frac{r_2}{r_1}e^{-4(r_1+r_2)/a}\left[(r_1+r_2)-|r_1-r_2|\right]$$

$$=2\pi\int_0^{r_1}\frac{2r_2^2}{r_1}e^{-4(r_1+r_2)/a}dr_2+2\pi\int_{r_1}^{\infty}2r_2e^{-4(r_1+r_2)/a}dr_2=$$

$$=\frac{4\pi e^{-4r_1/a}}{r_1}\underbrace{\int_0^{r_1}r_2^2e^{-4r_2/a}dr_2}_{I_1}+4\pi e^{-4r_1/a}\underbrace{\int_{r_1}^{\infty}r_2e^{-4r_2/a}dr_2}_{I_2}=$$

$$I_1:\ \int_0^{r_1}r_2^2e^{-4r_2/a}dr_2=\left[-\frac{a}{4}r_2^2e^{-4r_2/a}+\frac{a}{2}\left(\frac{a}{4}\right)^2e^{-4r_2/a}\left(-\frac{4r_2}{a}-1\right)\right]_0^{r_1}=$$

$$=-\frac{a}{4}\left[r_1^2e^{-4r_1/a}+\frac{ar_1}{2}e^{-4r_1/a}+\frac{a^2}{8}e^{-4r_1/a}-\frac{a^2}{8}\right]$$

$$\int_{r_1}^{\infty}r_2e^{-4r_2/a}dr_2=\left[\left(\frac{a}{4}\right)^2e^{-4r_2/a}\left(-\frac{4r_2}{a}-1\right)\right]_{r_1}^{\infty}=\left(\frac{a}{4}\right)^2e^{-4r_1/a}\left(\frac{4r_1}{a}+1\right)$$

Således

$$\frac{4\pi}{r_1}e^{-4r_1/a}(I_1+I_2)=\frac{\pi a^2}{8}\left\{\frac{a}{r_1}e^{-4r_1/a}-\left(2+\frac{a}{r_1}\right)e^{-8r_1/a}\right\}$$

$$\Rightarrow\left\langle\frac{1}{|r_1-r_2|}\right\rangle=\frac{8}{\pi a^4}\cdot4\pi\int_0^\infty\left[\frac{a}{r_1}e^{-4r_1/a}-\left(2+\frac{a}{r_1}\right)e^{-8r_1/a}\right]r_1^2dr_1=$$

$$=\frac{32}{a^4}\left\{a\int_0^\infty r_1e^{-4r_1/a}dr_1-2\int_0^\infty r_1^2e^{-8r_1/a}dr_1-a\int_0^\infty r_1e^{-8r_1/a}dr_1\right\}$$

$$=\frac{32}{a^4}\left\{a\cdot\left(\frac{a}{4}\right)^2-2\cdot2\cdot\left(\frac{a}{8}\right)^3-a\cdot\left(\frac{a}{8}\right)^2\right\}=\frac{32}{a}\left(\frac{1}{16}-\frac{1}{128}-\frac{1}{64}\right)=\frac{5}{4a}$$

b) Uppskatta elektronens interaktionsenergi

$$V_{ee} \approx \frac{e^2}{4\pi\varepsilon_0}\left\langle \frac{1}{|r_1-r_2|}\right\rangle = \frac{5}{4}\frac{e^2}{4\pi\varepsilon_0}\cdot\frac{1}{a} = \frac{5}{4}\frac{m}{\hbar^2}\left(\frac{e^2}{4\pi\varepsilon_0}\right)^2 = \frac{5}{2}(-E_1) = \frac{5}{2}(13.6\,eV) = 34$$