Introduction to Neutrino Physics

Tommy Ohlsson
tommy@theophys.kth.se

Royal Institute of Technology (KTH)
Stockholm Center for Physics, Astronomy, and Biotechnology
Stockholm, Sweden

Stockholms centrum för fysik astronomi bioteknik

Abisko, Norrbotten, Sweden – August 5-15, 2002
ν News

Neutrino news from ICHEP 2002, Amsterdam, The Netherlands, July 24-31, 2002 (where this lecture was prepared):

- The results of the LSND experiment will be tested by the MiniBooNE experiment, which will start in August 2002.
- CPT violation was surprisingly discussed a lot in the theory talks as an alternative to neutrino oscillations in order to also accommodate the results of the LSND experiment.
- KamLAND: First results @ PANIC 2002, Osaka, Japan
Brief History & Introduction

- **4 December 1930**: Wolfgang Pauli postulated the neutrino.
- **1933**: Enrico Fermi and Francis Perrin concluded that neutrinos are massless particles.
- **1946**: Bruno Pontecorvo proposed that neutrinos could be detected through the reaction \( \bar{\nu}_e + ^{37}\text{Cl} \rightarrow e^- + ^{37}\text{Ar} \).
- **1956**: Clyde L. Cowan, Jr. and Frederick Reines discovered the electron antineutrino via inverse \( \beta \)-decay, \( \bar{\nu}_e + p \rightarrow e^+ + n \). **Nobel Prize**
- **1957**: Neutrino oscillations were considered by Pontecorvo.
Brief History & Introduction

- **1962**: Mixing of two massive neutrinos was introduced and discussed by Z. Maki, M. Nakagawa, and S. Sakata.
- **1962**: The muon neutrino was detected by the group of L.M. Lederman, M. Schwartz, and J. Steinberger at the Brookhaven National Laboratory.  
  Nobel Prize
- **1978 & 1985**: The Mikheyev–Smirnov–Wolfenstein (MSW) effect was found.
- **23 February 1987**: About 25 neutrinos from the supernova SN1987A were detected by the Kamiokande (Japan), IMB (USA), and Baksan (Sovjet Union) experiments.
Brief History & Introduction

- **June 1998:** The Super-Kamiokande experiment in Japan reported strong evidence for neutrino oscillations from their atmospheric neutrino data and from which one can conclude that neutrinos are massive particles.

- **July 2000:** The tau neutrino was observed by the DONUT experiment at Fermilab.

- **June 2001:** The SNO experiment in Canada reported the first direct indication of a non-electron flavor component in the solar neutrino flux, *i.e.*, the first strong evidence for solar neutrino oscillations.
Brief History & Introduction

- **April 2002:** Continuation of the success of the SNO experiment, which reported its first results on the $^8\text{B}$ solar neutrino flux measurements via neutral-current (NC) interactions. The total neutrino flux measured via NC interactions is consistent with the Standard Solar Model (SSM) predictions.
Sources of Neutrinos

- The Sun
- The atmosphere (cosmic rays)
- Reactors
- Accelerators
- Supernovae
- The Earth ("natural radioactivity")
- The Big Bang (so-called relic neutrinos)

...
Neutrino Flavors

What is the number of neutrino flavors?

---

*a*Particle Data Group, K. Hagiwara et al., Review of Particle Physics, Phys. Rev. D 66, 010001 (2002), pdg.lbl.gov
Neutrino Flavors

What is the number of neutrino flavors?

A combined SM fit to LEP data \( ^{a} \):

\[
N_{\nu} = 2.994 \pm 0.012
\]

A direct measurement of invisible \( Z^0 \) decay width:

\[
N_{\nu} = 2.92 \pm 0.07
\]

\(^a\)Particle Data Group, K. Hagiwara \textit{et al.}, Review of Particle Physics, Phys. Rev. D \textbf{66}, 010001 (2002), \texttt{pdg.lbl.gov}
Neutrino Flavors

What is the number of neutrino flavors?

A combined SM fit to LEP data \(^a\):

\[ N_\nu = 2.994 \pm 0.012 \]

A direct measurement of invisible $Z^0$ decay width:

\[ N_\nu = 2.92 \pm 0.07 \]

\[ \therefore N_\nu = 3, \text{ i.e., there are three neutrino flavors.} \]

\(^a\)Particle Data Group, K. Hagiwara et al., Review of Particle Physics, Phys. Rev. D 66, 010001 (2002), pdg.lbl.gov
Massive Neutrinos

Within the standard model of elementary particle physics (SM), the neutrinos are massless particles.

⇒ Massive neutrinos would mean physics beyond the SM.

Neutrinos are most certain massive particles.
⇒ The SM needs to be extended!
Massive Neutrinos

The fermionic sector of the SM contains 13 parameters, \textit{i.e.},

1. 9 mass parameters (quarks and charged leptons) and
2. 4 mixing parameters (CKM mixing matrix).

A trivial extension of the SM would mean 7 (Dirac case) or 9 (Majorana case) new parameters, \textit{i.e.},

1. 3 mass parameters (neutrinos) and
2. 4 or 6 (leptonic) mixing parameters (MNS mixing matrix).

\therefore In total, the fermionic sector of the extended SM has 20 or 22 parameters.
Massive Neutrinos

One possible extension of the SM:
Simplest new physics (NP): Add new fields to the SM. The trivial extension is to add $n$ sterile neutrinos (right-handed SM singlets) to the three active neutrinos $[\nu_L = (\nu_{\text{e}L} \quad \nu_{\mu L} \quad \nu_{\tau L})^T]$. 

$\Rightarrow$ Two types of mass terms arise from renormalizable terms:

$$\mathcal{L}_{\text{NP}} = -\overline{\nu}_L M_D \nu_R - \frac{1}{2} \overline{\nu}_R^c M_R \nu_R + \text{h.c.}, \quad \nu_R = (\nu_{1R} \quad \nu_{2R} \cdots \quad \nu_{nR})^T,$$

which can be written as

$$\mathcal{L}_{\nu} = -\frac{1}{2} \overline{\nu}^c M_{\nu} \nu + \text{h.c.}, \quad \nu = \begin{pmatrix} \nu_L \\ \nu_R \end{pmatrix}, \quad M_{\nu} = \begin{pmatrix} 0 & M_D \\ M_D^T & M_R \end{pmatrix},$$

where $M_{\nu}$ is the $(3 + n) \times (3 + n)$ Dirac–Majorana neutrino mass matrix.
The Seesaw Mechanism

The *seesaw mechanism* is one of the most natural descriptions of the smallness of the neutrino masses.
The Seesaw Mechanism

The *seesaw mechanism* is one of the most natural descriptions of the smallness of the neutrino masses. One flavor Dirac–Majorana neutrino mass matrix:

\[
M = \begin{pmatrix}
    m_L & m_D \\
    m_D & m_R
\end{pmatrix},
\]

where \(m_L, m_R,\) and \(m_D\) are real mass parameters.
The Seesaw Mechanism

The *seesaw mechanism* is one of the most natural descriptions of the smallness of the neutrino masses. One flavor Dirac–Majorana neutrino mass matrix:

\[
M = \begin{pmatrix}
m_L & m_D \\
m_D & m_R
\end{pmatrix},
\]

where \( m_L, m_R, \) and \( m_D \) are real mass parameters.

Diagonalization \([M' = O^T M O = \text{diag}(m'_1, m'_2)]\) \( \Rightarrow \)

\[
m'_{1,2} = \frac{m_L + m_R}{2} \mp \frac{1}{2} \sqrt{(m_L - m_R)^2 + 4m_D^2}
\]
The Seesaw Mechanism

Assume that $m_L = 0$ and $m_D \ll m_R \quad \Rightarrow$

$$\begin{aligned}
m_1 &\equiv -m'_1 \simeq \frac{m_D^2}{m_R} \\
m_2 &\equiv m'_2 \simeq m_R
\end{aligned}$$

which are called the *seesaw mechanism formulas*.

(Gell-Mann, Ramond & Slansky, 1979; Yanagida, 1979; Mohapatra & Senjanović, 1980)
The Seesaw Mechanism

Matrix generalization of the seesaw mechanism formulas for more than one flavor:

\[ M = \begin{pmatrix} 0 & M_D \\ M_D^T & M_R \end{pmatrix} \rightarrow \quad \mathcal{M} = U^T M U = \begin{pmatrix} M_1 & 0 \\ 0 & M_2 \end{pmatrix}, \]

where

\[
\begin{cases} 
M_1 \simeq -M_D M_R^{-1} M_D^T \equiv M_\nu \\
M_2 \simeq M_R
\end{cases}
\]

Here \( M_\nu \) is the effective neutrino mass matrix, \( M_D \) is the Dirac mass matrix, and \( M_R \) is the Majorana mass matrix. (see, e.g., Lindner, Ohlsson & Seidl, 2002)
The Masses of the Neutrinos

Assuming that the neutrino mass eigenstates $\nu_1$, $\nu_2$, and $\nu_3$ are the primary components of the neutrino flavor states $\nu_e$, $\nu_\mu$, and $\nu_\tau$, the present upper bounds on the neutrino masses are:

- The electron neutrino: $m_{\nu_e} < 3$ eV
- The muon neutrino: $m_{\nu_\mu} < 0.19$ MeV
- The tau neutrino: $m_{\nu_\tau} < 18.2$ MeV

However, this picture is not entirely true!
Neutrino Oscillations

Neutrino oscillations ⇔ Neutrinos are massive and mixed.
Neutrino Oscillations

Neutrino oscillations $\iff$ Neutrinos are massive and mixed.

The neutrinos are particles that can be considered to be governed by the *Schrödinger equation*, *i.e.*, the neutrino states can be represented by quantum mechanical states.
Neutrino Oscillations

Neutrino oscillations $\iff$ Neutrinos are massive and mixed.

The neutrinos are particles that can be considered to be governed by the Schrödinger equation, i.e., the neutrino states can be represented by quantum mechanical states.

The relations between flavor and mass states and fields are:

$$|\nu_\alpha\rangle = \sum_{a=1}^{N} U_{\alpha a}^* |\nu_a\rangle \iff \nu_\alpha = \sum_{a=1}^{N} U_{\alpha a} \nu_a \quad (\alpha = e, \mu, \tau, \ldots),$$

where the $U_{\alpha a}$'s are matrix elements of the mixing matrix $U$. 
Neutrino Oscillations

The time evolution of the neutrino mass eigenstates is:

$$|\nu_a(t)\rangle = e^{-iE_a t} |\nu_a(0)\rangle,$$

where $E_a = \sqrt{m_a^2 + p_a^2} \simeq p_a + \frac{m_a^2}{2p_a}$ is the energy of the $a$th neutrino mass eigenstate $\nu_a$ (ultra-relativistic neutrinos). Here $m_a$ and $p_a$ are the mass and momentum of the $a$th neutrino mass eigenstate, respectively.

Furthermore, we can assume that $p_a \simeq p_b \equiv p \simeq E$ and $t \simeq L$. 
Neutrino Oscillations

Thus, neutrino oscillations among different neutrino flavors occur with the transition probabilities

\[ P_{\alpha\beta} \equiv P(\nu_\alpha \rightarrow \nu_\beta) = \left| \langle \nu_\beta(L) | \nu_\alpha(0) \rangle \right|^2 \]

\[ = \left| \sum_{a=1}^{N} \sum_{b=1}^{N} U_{\alpha a}^* U_{\beta b} \langle \nu_b(L) | \nu_a(0) \rangle \right|^2 , \]

which can be re-written as

\[ P_{\alpha\beta} = \delta_{\alpha\beta} \]

\[ - 4 \sum_{a=1}^{N} \sum_{b=1}^{N} \Re(U_{\alpha a}^* U_{\beta a} U_{\alpha b} U_{\beta b}^*) \sin^2 \frac{\Delta m_{ab}^2 L}{4E} \]

\[ - 2 \sum_{a=1}^{N} \sum_{b=1}^{N} \Im(U_{\alpha a}^* U_{\beta a} U_{\alpha b} U_{\beta b}^*) \sin \frac{\Delta m_{ab}^2 L}{2E} , \quad \alpha, \beta = e, \mu, \tau, \ldots \]
The MSW Effect

The neutrino oscillation transition probabilities in matter may differ drastically from the corresponding transition probabilities in vacuum.

In particular, the presence of matter can lead to resonant amplification of the transition probability between two given types of neutrinos, even if the corresponding transition probability in vacuum is strongly suppressed by a small mixing.

This is the so-called MSW effect.

(Wolfenstein, 1978; Mikheyev & Smirnov, 1985, 1986)
Matter Effects

The Schrödinger equation in vacuum:

\[ i \frac{d}{dt} \psi_m(t) = \mathcal{H}_m \psi_m(t), \quad \psi_m = \begin{pmatrix} \psi_1 \\ \psi_2 \\ \vdots \\ \psi_N \end{pmatrix}, \]

where the total (free) Hamiltonian in the mass basis is

\[ \mathcal{H}_m = H_m = \text{diag} (E_1, E_2, \ldots, E_N). \]

Here \( N \) are the number of flavors and \( E_a = m_a^2/(2E) \).
Matter Effects

The Schrödinger equation in matter:
In matter, we need to add the matter potential to the total Hamiltonian, \(i.e.,\)

\[
\mathcal{H}_m = H_m = \text{diag} (E_1, E_2, \ldots, E_N)
\]

\[
\rightarrow
\]

\[
\mathcal{H}_m = H_m + U^{-1}V_f U
\]

\[
= \text{diag} (E_1, E_2, \ldots, E_N) + U^{-1} \text{diag} (A, 0, \ldots, 0) U,
\]

where \(U\) is the mixing matrix relating the mass and flavor bases and \(A\) is a constant, the matter density parameter.

Note! Only one entry in the matrix \(V_f\) is non-zero.
Two Flavor Neutrino Oscillation Models

In vacuum:

The mixing matrix:

where \( \theta \) is the mixing angle.

The mass squared difference:

where \( m_1 \) and \( m_2 \) are the masses of the two neutrino mass eigenstates, respectively.

MSW resonance condition:

The effective mixing angle in matter obtains its maximal value when the condition

is fulfilled. This condition is called the MSW resonance condition.

It follows that, if the MSW resonance condition is satisfied, then the effective mixing angle in matter is maximal, i.e., \( \theta_{\text{eff}} = \frac{1}{2} \sin(2\theta) \), independently of the vacuum mixing angle.
Two Flavor Neutrino Oscillation Models

In vacuum:

The mixing matrix:

\[ U = \begin{pmatrix} \cos \theta & \sin \theta \\ -\sin \theta & \cos \theta \end{pmatrix}, \]

where \( \theta \) is the mixing angle.
Two Flavor Neutrino Oscillation Models

In vacuum:

The mixing matrix:

\[ U = \begin{pmatrix} \cos \theta & \sin \theta \\ -\sin \theta & \cos \theta \end{pmatrix}, \]

where \( \theta \) is the mixing angle.

The mass squared difference:

\[ \Delta m^2 = m_2^2 - m_1^2, \]

where \( m_1 \) and \( m_2 \) are the masses of the two neutrino mass eigenstates, respectively.
Two Flavor Neutrino Oscillation Models

The transition probabilities:

\[ P_{\alpha\beta} = \delta_{\alpha\beta} - (2\delta_{\alpha\beta} - 1) \sin^2 2\theta \sin^2 \frac{\Delta m^2 L}{4E}, \]

where \( L \) is the baseline length and \( E \) is the neutrino energy.

It holds that \( P_{ee} = 1 - P_{e\mu} = 1 - P_{\mu e} = P_{\mu\mu} \).
Two Flavor Neutrino Oscillation Models

**In matter:**
The effective mixing angle in matter:

\[
\sin^2 2\theta^M \equiv \frac{\sin^2 2\theta}{\sin^2 2\theta + \left(\cos 2\theta - \frac{2E}{\Delta m^2 A}\right)^2}
\]

The effective mass squared difference in matter:

\[
\Delta \tilde{m}^2 \equiv \Delta m^2 \sqrt{\sin^2 2\theta + \left(\cos 2\theta - \frac{2E}{\Delta m^2 A}\right)^2}
\]

Here \( A \) is the matter density parameter.

\( A = 0 \) \( \Rightarrow \) \( \theta^M = \theta \) and \( \Delta \tilde{m}^2 = \Delta m^2 \)
Two Flavor Neutrino Oscillation Models

MSW resonance condition:
Two Flavor Neutrino Oscillation Models

MSW resonance condition:
The effective mixing angle in matter $\theta^M$ obtains its maximal value $\sin^2 2\theta^M = 1$ when the condition

$$\cos 2\theta = \frac{2E}{\Delta m^2} A$$

is fulfilled. This condition is called the MSW resonance condition.
Two Flavor Neutrino Oscillation Models

**MSW resonance condition:**

The effective mixing angle in matter $\theta^M$ obtains its maximal value $\sin^2 2\theta^M = 1$ when the condition

$$\cos 2\theta = \frac{2E}{\Delta m^2 A}$$

is fulfilled. This condition is called the *MSW resonance condition.*

It follows that, if the MSW resonance condition is satisfied, then the effective mixing angle in matter $\theta^M$ is maximal, *i.e.*, $\theta^M = 45^\circ$, independently of the value of the vacuum mixing angle $\theta$. 
Three Flavor Neutrino Oscillation Models

The formulas in matter are much more complicated for the three flavor case than for the two flavor case!
Three Flavor Neutrino Oscillation Models

The formulas in matter are much more complicated for the three flavor case than for the two flavor case!

Using the Cayley–Hamilton formalism, one finds the transition probabilities in matter:

\[
P_{\alpha\beta} = \delta_{\alpha\beta} - 4 \sum_{a=1}^{3} \sum_{b=1}^{3} \sum_{a<b} \frac{(\lambda_a^2 + c_1)\delta_{\alpha\beta} + \lambda_a \tilde{T}_{\beta\alpha} + (\tilde{T}^2)_{\beta\alpha}}{3\lambda_a^2 + c_1} \sin^2 \tilde{x}_{ab},
\]

where \(\tilde{x}_{ab} \equiv (\lambda_a - \lambda_b)L/2\).
Three Flavor Neutrino Oscillation Models

Here:

\[ T = \mathcal{H}_m - \frac{1}{3} (\text{tr } \mathcal{H}_m) 1_3 \]

\[ \text{tr } \mathcal{H}_m = E_1 + E_2 + E_3 + A \]

\[ c_1 = \det T \text{tr } T^{-1} \]

\[ \tilde{T} = U T U^{-1} \]

The \( \lambda_a \)'s \( (a = 1, 2, 3) \) are the eigenvalues of the matrix \( T \).

(Ohlsson & Snellman, 2000; Ohlsson, 2001)
Three Flavor Neutrino Oscillation Models

The survival transition probability $P_{ee}$ for neutrinos that traverse the Earth as a function of the nadir angle $h$ and the neutrino energy $E$:  

\[(\text{Ohlsson, 2001})\]

Parameter values: $\theta_{12} = \theta_{23} = 45^\circ$, $\theta_{13} = 5^\circ$, $\delta_{\text{CP}} = 0$, $\Delta M^2 = 3.2 \cdot 10^{-3}$ eV$^2$, and $\Delta m^2 = \Delta M^2 / 10$. 
Neutrino Experiments

Other sources of neutrinos:

- $e^- + ^7\text{Be} \rightarrow ^7\text{Li} + \nu_e$
- $^8\text{B} \rightarrow ^7\text{He} + e^+ + \nu_e$
- Cosmic-ray shower
  - $\pi^+ \rightarrow \mu^+ + \nu_\mu$
  - $\pi^- \rightarrow \mu^- + \bar{\nu}_\mu$
- Atmospheric neutrino source
  - $\pi^+ \rightarrow e^+ + \nu_e$
  - $\pi^- \rightarrow e^- + \bar{\nu}_e$

Earth

Underground $\nu_e$ detector

Solar core

Primary neutrino source $p + p \rightarrow D + e^+ + \nu_e$

Atmospheric neutrino source $\pi^+ \rightarrow \mu^+ + \nu_\mu$

Atmospheric neutrino source $\pi^- \rightarrow \mu^- + \bar{\nu}_\mu$

Atmospheric neutrino source

Neutrinos emitted from the sun

Tommy Ohlsson - Abisko 2002 – p.14/25
Neutrino Experiments

Five different types of neutrino oscillation experiments: solar, atmospheric, accelerator, reactor, and cosmic.

Old experiments: Homestake, SAGE, GALLEX, Kamiokande (solar); Baksan, NUSEX, Fréjus, IMB, MACRO, Soudan, Kamiokande (atmospheric); NOMAD, CHORUS, LSND, KARMEN (accelerator); Bugey, CHOOZ, Gösgen, Palo Verde (reactor)
Neutrino Experiments

Present and new experiments:

- Solar: Super-Kamiokande, Hyper-Kamiokande, SNO, GNO, BOREXINO
- Atmospheric: Super-Kamiokande, Hyper-Kamiokande, MONOLITH
- Reactor: KamLAND
- Accelerator LBL: K2K, MINOS, CERN-LNGS, MiniBooNE, BooNE
- Cosmic: AMANDA, Baikal, Antares, IceCube

In the order of magnitude of 10 years: Neutrino factory
The Solar Neutrino Problem

The Sun is a source of neutrinos (solar neutrinos). The solar neutrinos can be detected on Earth using underground detectors. However, the measured fluxes of the solar neutrinos and the theoretical estimates for the solar neutrino fluxes are different. The measured flux is about one-half of the “expected” flux. This is the solar neutrino problem.
The Solar Neutrino Problem

The Sun is a source of neutrinos (solar neutrinos). The solar neutrinos can be detected on Earth using underground detectors. However, the measured fluxes of the solar neutrinos and the theoretical estimates for the solar neutrino fluxes are different. The measured flux is about one-half of the “expected” flux. This is the solar neutrino problem.

SSM (BP2000): (Bahcall, Pinsonneault & Basu, 2001)
The Solar Neutrino Problem

Experimental results:  (Bilenky, Giunti & Grimus, 1999)

<table>
<thead>
<tr>
<th>Experiment</th>
<th>Ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>Homestake</td>
<td>$0.33^{+0.06}_{-0.05}$</td>
</tr>
<tr>
<td>SAGE</td>
<td>$0.52 \pm 0.07$</td>
</tr>
<tr>
<td>GALLEX</td>
<td>$0.60 \pm 0.07$</td>
</tr>
<tr>
<td>Kamiokande</td>
<td>$0.54 \pm 0.07$</td>
</tr>
<tr>
<td>Super-Kamiokande</td>
<td>$0.47^{+0.07}_{-0.09}$</td>
</tr>
</tbody>
</table>
The Solar Neutrino Problem

Experimental results: (Bilenky, Giunti & Grimus, 1999)

<table>
<thead>
<tr>
<th>Experiment</th>
<th>Ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>Homestake</td>
<td>0.33±0.06</td>
</tr>
<tr>
<td>SAGE</td>
<td>0.52 ± 0.07</td>
</tr>
<tr>
<td>GALLEX</td>
<td>0.60 ± 0.07</td>
</tr>
<tr>
<td>Kamiokande</td>
<td>0.54 ± 0.07</td>
</tr>
<tr>
<td>Super-Kamiokande</td>
<td>0.47±0.07</td>
</tr>
</tbody>
</table>

Solution: Neutrino oscillations

(Best channels: $\nu_e \to \nu_{\mu,\tau}$ with $\Delta m^2_\odot \simeq 5.0 \cdot 10^{-5} \text{eV}^2$ and $\theta_\odot \simeq 33^\circ$ [LMA])
The Solar Neutrino Problem

SNO:  (Ahmad et al., 2001, 2002)

CC:  \( \nu_e + d \rightarrow e^- + p + p, \quad E_{\text{th}} \simeq 1.44 \text{ MeV}, \)

NC:  \( \nu_\alpha + d \rightarrow \nu_\alpha + p + n, \quad \alpha = e, \mu, \tau, \quad E_{\text{th}} \simeq 2.23 \text{ MeV}, \)

ES:  \( \nu_\alpha + e^- \rightarrow \nu_\alpha + e^-, \quad \alpha = e, \mu, \tau. \)
The Solar Neutrino Problem

The Sun as seen by the Super-Kamiokande detector!

**Note!** The detector is located in an old gold mine a few kilometers below the surface of the Earth. Sun-light will never reach the detector, but solar neutrinos will.
Atmospheric neutrinos are produced in the following reaction chain:

cosmic rays + air \rightarrow \pi^{\pm}(K^{\pm}) + X

\pi^{\pm}(K^{\pm}) \rightarrow \mu^{\pm} + \nu_{\mu}/\overline{\nu}_{\mu}

\mu^{\pm} \rightarrow e^{\pm} + \nu_{e}/\overline{\nu}_{e} + \overline{\nu}_{\mu}/\nu_{\mu}.
Atmospheric neutrinos are produced in the following reaction chain:

\[
\text{cosmic rays + air} \rightarrow \pi^\pm(K^\pm) + X
\]

\[
\pi^\pm(K^\pm) \rightarrow \mu^\pm + \nu_\mu/\bar{\nu}_\mu
\]

\[
\mu^\pm \rightarrow e^\pm + \nu_e/\bar{\nu}_e + \bar{\nu}_\mu/\nu_\mu.
\]

Naively, one concludes from the above reaction chain that

\[
R(\nu_\mu/\nu_e) \equiv \frac{N_{\nu_\mu} + N_{\bar{\nu}_\mu}}{N_{\nu_e} + N_{\bar{\nu}_e}} = 2.
\]

The “ratio of ratios”:

\[
\mathcal{R} \equiv \frac{R(\nu_\mu/\nu_e)_{\text{exp.}}}{R(\nu_\mu/\nu_e)_{\text{MC}}}
\]
The Atmospheric Neutrino Problem

\[ R \neq 1 \iff \text{Atmospheric neutrino problem} \]
The Atmospheric Neutrino Problem

\[ R \neq 1 \iff \text{Atmospheric neutrino problem} \]

Experimental results:

![Diagram showing experimental results for atmospheric neutrino problem]

- NUSEX
- SOUDAN2
- IMB
- FREJUS
- KAM multi
- KAM sub
- SK multi
- SK sub

Tommy Ohlsson - Abisko 2002 – p.16/25
The Atmospheric Neutrino Problem

\[ \mathcal{R} \neq 1 \iff \text{Atmospheric neutrino problem} \]

Experimental results:

\[ \frac{R_{\mu/e}}{R_{\nu/e}} \]

**Solution:** Neutrino oscillations

(Best channel: \( \nu_\mu \rightarrow \nu_\tau \) with \( \Delta m^2_{\text{atm}} \approx 2.5 \times 10^{-3} \text{ eV}^2 \) and \( \theta_{\text{atm}} \approx 45^\circ \))
The Atmospheric Neutrino Problem

Super-Kamiokande (1289 days): (Toshito, 2001)
The Accelerator SBL (LSND) Neutrino Problem

The only positive signature of neutrino oscillations at a laboratory experiment comes from the LSND experiment at Los Alamos.
The only positive signature of neutrino oscillations at a laboratory experiment comes from the LSND experiment at Los Alamos. However, only $87.9 \pm 22.4 \pm 6.0$ events, which corresponds to

$$P(\bar{\nu}_\mu \rightarrow \bar{\nu}_e) = (0.264 \pm 0.067 \pm 0.045)\%$$

with

$$\Delta m_{\text{LSND}}^2 \in (0.2, 10) \text{ eV}^2.$$  

(Aguilar et al., 2001)
The Accelerator SBL (LSND) Neutrino Problem

The only positive signature of neutrino oscillations at a laboratory experiment comes from the LSND experiment at Los Alamos. However, only $87.9 \pm 22.4 \pm 6.0$ events, which corresponds to

$$P(\bar{\nu}_\mu \rightarrow \bar{\nu}_e) = (0.264 \pm 0.067 \pm 0.045)\% \text{ with } \Delta m^2_{\text{LSND}} \in (0.2, 10) \text{ eV}^2.$$  

(Aguilar et al., 2001)

**Problem:** A new mass squared difference, $\Delta m^2_{\text{LSND}}$.

$$\Delta m^2_{\odot} \ll \Delta m^2_{\text{atm}} \ll \Delta m^2_{\text{LSND}}$$

$\Rightarrow$ More than three flavors is necessary!
The Accelerator SBL (LSND) Neutrino Problem

The only positive signature of neutrino oscillations at a laboratory experiment comes from the LSND experiment at Los Alamos. However, only $87.9 \pm 22.4 \pm 6.0$ events, which corresponds to $P(\bar{\nu}_\mu \rightarrow \bar{\nu}_e) = (0.264 \pm 0.067 \pm 0.045)\%$ with $\Delta m^2_{\text{LSND}} \in (0.2, 10) \text{ eV}^2$. (Aguilar et al., 2001)

Problem: A new mass squared difference, $\Delta m^2_{\text{LSND}}$.

$$\Delta m^2_\odot \ll \Delta m^2_{\text{atm}} \ll \Delta m^2_{\text{LSND}}$$

$\Rightarrow$ More than three flavors is necessary!

Solution: The LSND experiment is wrong! (easiest and most possible); sterile neutrinos, CPT violation, ... (not so possible)
The Accelerator SBL (LSND) Neutrino Problem

The KARMEN experiment has excluded most of the region of the parameter space, which is favored by the LSND experiment.

The results of the LSND experiment will be further tested by the MiniBooNE experiment, which starts running in August 2002.
Results from Analyses

Solar neutrinos: $\nu_e - \nu_{\mu,\tau}$ oscillations

LMA: $\Delta m^2_\odot \approx 5.0 \cdot 10^{-5} \text{ eV}^2$ and $\theta_\odot \approx 33^\circ$

(Bahcall, Gonzalez-Garcia & Peña-Garay, 2001, 2002)

Further tests: KamLAND experiment

Atmospheric neutrinos: $\nu_\mu - \nu_\tau$ oscillations

$\Delta m^2_{\text{atm}} \approx 2.5 \cdot 10^{-3} \text{ eV}^2$ and $\theta_{\text{atm}} \approx 45^\circ$

(Fukuda et al., 1998; Toshito, 2001; Shiozawa, 2002)

Further tests: Long-baseline experiments
Results from Analyses

**LSND experiment:** $\bar{\nu}_\mu - \bar{\nu}_e$ oscillations

$$\Delta m^2_{\text{LSND}} \in (0.2, 2) \text{ eV}^2 \quad \text{and} \quad \theta_{\text{LSND}} \in (1, 6)^\circ$$

(Athanassopoulos et al., 1996, 1998; Aguilar et al., 2001)

**Further tests:** MiniBooNE experiment

**Reactor neutrinos (CHOOZ experiment):**

$$\theta_{\text{CHOOZ}} \lesssim 9.2^\circ$$

(Apollonio et al., 1999)

$\Rightarrow$ Three flavor models decouple into two two flavor models.
Conclusions!

- Neutrino mixing has moved from a speculation to an observed property of nature.
- For the simplest model we have measured 2 of the 3 angles, 3 of the 2 mass differences, and the sign of one mass difference.
- Results over the next few years will be of great interest, with SNO, KamLAND, MiniBooNE, K2K, BOREXINO, MINOS, CNGS, and others all contributing new data.
- The farther future leads from Superbeams through the Neutrino Factory, and we can look forward to new discoveries at each step.
- Direct mass measurements, including double-beta decay, must be a priority.
- There is much work to be done – JOIN US!
- Thanks to everybody who I stole transparencies from!
Neutrino Oscillation Parameters

Definition (neutrino mass squared differences):

\[ \Delta m_{ab}^2 = m_a^2 - m_b^2, \]

where \( m_a \) \((a = 1, 2, 3)\) is the mass of the \( a \)th neutrino mass eigenstate.
Neutrino Oscillation Parameters

Definition (neutrino mass squared differences):

\[ \Delta m_{ab}^2 = m_a^2 - m_b^2, \]

where \( m_a \) (\( a = 1, 2, 3 \)) is the mass of the \( a \)th neutrino mass eigenstate.

Neutrino mass squared differences:

- \( \Delta m^2 \equiv \Delta m_{21}^2 \): The small (or “solar”) mass squared difference
- \( \Delta M^2 \equiv \Delta m_{32}^2 \sim \Delta m_{31}^2 \): The large (or “atmospheric”) mass squared difference
Neutrino Oscillation Parameters

Definition (neutrino mass squared differences):

\[ \Delta m_{ab}^2 = m_a^2 - m_b^2, \]

where \( m_a \) \((a = 1, 2, 3)\) is the mass of the \( a \)th neutrino mass eigenstate.

Neutrino mass squared differences:

- \( \Delta m^2 \equiv \Delta m_{21}^2 \): The small (or “solar”) mass squared difference
- \( \Delta M^2 \equiv \Delta m_{32}^2 \sim \Delta m_{31}^2 \): The large (or “atmospheric”) mass squared difference

Consequence: \( \Delta m_{21}^2 + \Delta m_{32}^2 + \Delta m_{13}^2 = 0 \)
Neutrino Oscillation Parameters

Leptonic mixing parameters:

- $\theta_{12} \equiv \theta_3$: The “solar” mixing angle
- $\theta_{13} \equiv \theta_2$: The “reactor” mixing angle
- $\theta_{23} \equiv \theta_1$: The “atmospheric” mixing angle
- $\delta_{CP} \equiv \delta$: The (leptonic) CP violation phase
Neutrino Oscillation Parameters

Leptonic mixing parameters:

- $\theta_{12} \equiv \theta_3$: The “solar” mixing angle
- $\theta_{13} \equiv \theta_2$: The “reactor” mixing angle
- $\theta_{23} \equiv \theta_1$: The “atmospheric” mixing angle
- $\delta_{\text{CP}} \equiv \delta$: The (leptonic) CP violation phase

The “standard” parameterization:

$$U = \begin{pmatrix}
C_2 C_3 & S_3 C_2 & S_2 e^{-i\delta} \\
-S_3 C_1 - S_1 S_2 C_3 e^{i\delta} & C_1 C_3 - S_1 S_2 S_3 e^{i\delta} & S_1 C_2 \\
S_1 S_3 - S_2 C_1 C_3 e^{i\delta} & -S_1 C_3 - S_2 S_3 C_1 e^{i\delta} & C_1 C_2
\end{pmatrix},$$

where $S_a \equiv \sin \theta_a$ and $C_a \equiv \cos \theta_a$ (for $a = 1, 2, 3$).
Neutrino Oscillation Parameters

\[ \therefore \text{2 neutrino mass squared differences and 4 leptonic mixing parameters, \textit{i.e.}, in total 6 neutrino oscillation parameters.} \]

**Note!** Two neutrino flavors: 1 mass squared difference and 1 leptonic mixing parameter.
Neutrino Oscillation Parameters

Transformation of any $3 \times 3$ unitary matrix to the “standard” parameterization: \cite{Ohlsson & Seidl, 2002}

$$
U = \begin{pmatrix}
U_{11} & U_{12} & U_{13} \\
U_{21} & U_{22} & U_{23} \\
U_{31} & U_{32} & U_{33}
\end{pmatrix} \rightarrow U = U(\theta_{12}, \theta_{13}, \theta_{23}, \delta_{CP})
$$
Neutrino Oscillation Parameters

Transformation of any $3 \times 3$ unitary matrix to the “standard” parameterization: (Ohlsson & Seidl, 2002)

$$U = \begin{pmatrix} U_{11} & U_{12} & U_{13} \\ U_{21} & U_{22} & U_{23} \\ U_{31} & U_{32} & U_{33} \end{pmatrix} \rightarrow U = U(\theta_{12}, \theta_{13}, \theta_{23}, \delta_{CP})$$

$\Rightarrow$ The parameters of the “standard” parameterization:

$$\theta_{12} = \text{arctan} \left| \frac{U_{12}}{U_{11}} \right|,$$

$$\theta_{13} = \text{arcsin} |U_{13}|,$$

$$\theta_{23} = \text{arctan} \left| \frac{U_{23}}{U_{33}} \right|,$$

$$\delta_{CP} = \text{arg} U_{11} + \text{arg} U_{12} - \text{arg} U_{13} + \text{arg} U_{23} + \text{arg} U_{33}.$$
### Present Values of the Neutrino Oscillation Parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Best-fit value</th>
<th>Range</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Delta m^2$</td>
<td>$5 \cdot 10^{-5}$ eV$^2$</td>
<td>$(2.3 \div 37) \cdot 10^{-5}$ eV$^2$ (99.73% CL)</td>
</tr>
<tr>
<td>$\Delta M^2$</td>
<td>$2.5 \cdot 10^{-3}$ eV$^2$</td>
<td>$(1.6 \div 3.9) \cdot 10^{-3}$ eV$^2$ (90% CL)</td>
</tr>
<tr>
<td>$\theta_{12}$</td>
<td>$33^\circ$</td>
<td>$26^\circ \div 43^\circ$ (99.73% CL)</td>
</tr>
<tr>
<td>$\theta_{13}$</td>
<td>-</td>
<td>$0 \div 9.2^\circ$</td>
</tr>
<tr>
<td>$\theta_{23}$</td>
<td>$45^\circ$</td>
<td>$37^\circ \div 45^\circ$ (90% CL)</td>
</tr>
<tr>
<td>$\delta_{\text{CP}}$</td>
<td>-</td>
<td>-</td>
</tr>
</tbody>
</table>

Note! Leptons (MNS): 2 large mixing angles and 1 small mixing angle.

Quarks (CKM): 3 small mixing angles.
### Present Values of the Neutrino Oscillation Parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Best-fit value</th>
<th>Range</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Delta m^2$</td>
<td>$5 \cdot 10^{-5}$ eV$^2$</td>
<td>$(2.3 \div 37) \cdot 10^{-5}$ eV$^2$ (99.73% CL)</td>
</tr>
<tr>
<td>$\Delta M^2$</td>
<td>$2.5 \cdot 10^{-3}$ eV$^2$</td>
<td>$(1.6 \div 3.9) \cdot 10^{-3}$ eV$^2$ (90% CL)</td>
</tr>
<tr>
<td>$\theta_{12}$</td>
<td>$33^\circ$</td>
<td>$26^\circ \div 43^\circ$ (99.73% CL)</td>
</tr>
<tr>
<td>$\theta_{13}$</td>
<td>-</td>
<td>$0 \div 9.2^\circ$</td>
</tr>
<tr>
<td>$\theta_{23}$</td>
<td>$45^\circ$</td>
<td>$37^\circ \div 45^\circ$ (90% CL)</td>
</tr>
<tr>
<td>$\delta_{CP}$</td>
<td>-</td>
<td>-</td>
</tr>
</tbody>
</table>

∴ **Bilarge leptonic mixing, i.e., $\theta_{12}$ and $\theta_{23}$ are large and $\theta_{13}$ is small.**
## Present Values of the Neutrino Oscillation Parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Best-fit value</th>
<th>Range</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Delta m^2$</td>
<td>$5 \cdot 10^{-5} \text{ eV}^2$</td>
<td>$(2.3 \div 37) \cdot 10^{-5} \text{ eV}^2$ (99.73% CL)</td>
</tr>
<tr>
<td>$\Delta M^2$</td>
<td>$2.5 \cdot 10^{-3} \text{ eV}^2$</td>
<td>$(1.6 \div 3.9) \cdot 10^{-3} \text{ eV}^2$ (90% CL)</td>
</tr>
<tr>
<td>$\theta_{12}$</td>
<td>$33^\circ$</td>
<td>$26^\circ \div 43^\circ$ (99.73% CL)</td>
</tr>
<tr>
<td>$\theta_{13}$</td>
<td>$-$</td>
<td>$0 \div 9.2^\circ$</td>
</tr>
<tr>
<td>$\theta_{23}$</td>
<td>$45^\circ$</td>
<td>$37^\circ \div 45^\circ$ (90% CL)</td>
</tr>
<tr>
<td>$\delta_{CP}$</td>
<td>$-$</td>
<td>$-$</td>
</tr>
</tbody>
</table>

\[ \therefore \quad \text{Bilarge leptonic mixing, i.e., } \theta_{12} \text{ and } \theta_{23} \text{ are large and } \theta_{13} \text{ is small.} \]

**Note!**

Leptons (MNS): 2 *large* mixing angles and 1 *small* mixing angle

Quarks (CKM): 3 *small* mixing angles
Other Scenarios

- Neutrino decay
  (see, e.g., Lindner, Ohlsson & Winter, 2001, 2002)

- Neutrino decoherence
  (see, e.g., Ohlsson, 2001)

- Spin-flavor precession
  (Lim & Marciano, 1988; Akhmedov, 1988)

- Flavor changing neutrino interactions

- Non-standard neutrino interactions

- CPT violation
  (see, e.g., Bilenky, Freund, Lindner, Ohlsson & Winter, 2002)

...
Neutrinos in Astrophysics

What is the absolute neutrino mass scale?
Neutrinos in Astrophysics

What is the absolute neutrino mass scale?

SN1987A

SN1987A data $\Rightarrow m_{\nu_e} < 20$ eV
Neutrinos in Astrophysics

What is the absolute neutrino mass scale?

SN1987A

SN1987A data $\Rightarrow m_{\nu_e} < 20$ eV

Future galactic supernovae (Super-Kamiokande) $\Rightarrow m_{\nu_e} \sim 3$ eV
Neutrinos in Astrophysics

The sum of the neutrino masses:

\[ M \equiv \sum_{a=1}^{3} m_a, \]

where the \( m_a \)'s are the masses of the neutrino mass eigenstates.

**Note!** This quantity is often used in astrophysics and cosmology.
Neutrinos in Astrophysics

From CMBR and large scale structure measurements:

\[ M < (2.5 \div 3) \text{ eV} \quad (@ 95\% \text{ C.L.}) \]

(Hannestad, 2002)

From CMBR, galaxy clustering, and Lyman \( \alpha \)-forest measurements:

\[ M < 4.2 \text{ eV} \quad (@ 95\% \text{ C.L.}) \]

(Wang, Tegmark & Zaldarriaga, 2002)

From the 2dF Galaxy Redshift Survey measurements:

\[ M < (1.8 \div 2.2) \text{ eV} \quad (@ 95\% \text{ C.L.}) \]

(Elgarøy et al., 2002)
Neutrinos in Astrophysics

In the future, combination of CMBR from MAP/Planck satellite with Sloan Digital Sky Survey measurements:

\[ M < 0.3 \text{ eV} \]

(Hu, Eisenstein & Tegmark, 1998)

Note!

An order of magnitude less than the present measurements!
Summary

✔ The number of light neutrino flavors are three.
✔ Neutrinos are massive and mixed.
✔ Neutrino oscillations occur among different flavors.
✔ The SM has to be extended in order to include massive neutrinos.
✔ New data soon from the following interesting experiments: KamLAND, MiniBooNE, . . .
✔ In the far future: long-baseline experiments and a neutrino factory. In the meantime, theoretical development!
Neutrino Physics in Progress

![Graph showing the number of papers per year containing the word "neutrino" in the title.]

Now: ~ 500 papers/year in neutrino physics!