Special Relativity summary

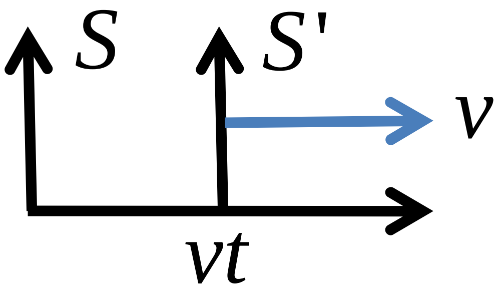
MW 090427

Postulates of Newtonian mechanics

- Invariance principles
  - Space is uniform: all laws are translation invariant
  - Space is isotropic: all laws are invariant under rotation
  - Galilean invariance: all laws are invariant under addition of a constant velocity (boost)
- Newtons laws
  - 1st law: inertial frames exist, where force free motion has constant velocity
  - 2nd law: $\boldsymbol{F}=m\boldsymbol{a}$ in inertial frames
  - 3rd law (law of action and reaction): $\boldsymbol{F}_{21}=-\boldsymbol{F}_{12}$

Results of Newtonian mechanics

- **There is no speed limit.**
  Information can be instantaneously transmitted.
- Galilean transformation between inertial systems S, S’ moving with relative speed v



$$\begin{cases} x = x' + vt \\ y = y' \\ z = z' \\ t = t' \end{cases}$$

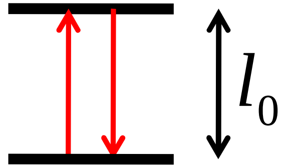
- Absolute space and time:
  space and time intervals are independent of the motion of the observer.
- Galilean velocity addition: $v_p = v'_p + v$

Special relativity:
a world with a speed limit

- Experimental facts
  1. The speed of light $c = 2.99792458 \ \text{m/s} \approx 3 \cdot 10^8 \ \text{m/s}$ is independent of the velocity of the light source
  2. Newtonian mechanics applies only in the nonrelativistic regime $v << c$
  3. Particles cannot be accelerated to speeds higher than the speed of light
     (hence Galilei velocity addition is wrong)
- **Einstein postulates of special relativity**
  1. **All laws are the same in all inertial frames**
  2. **There is a universal speed limit =*c***

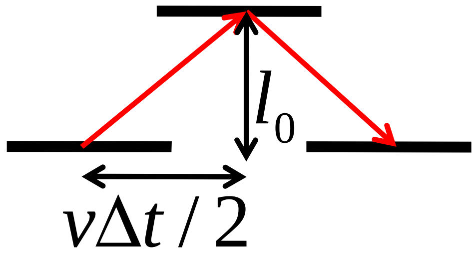
Time dilation, Lorentz factor: moving clocks run slow

- A simple clock (all clocks are equivalent):
  Let light bounce between mirrors
  time=(n.o. periods)*(period time)
- In the rest frame S’
  $t' = \tau$ is the proper time



$$\Delta t' = 2l_0 / c = \Delta\tau$$

- Seen from S, moving
  with speed *v* wrt S’

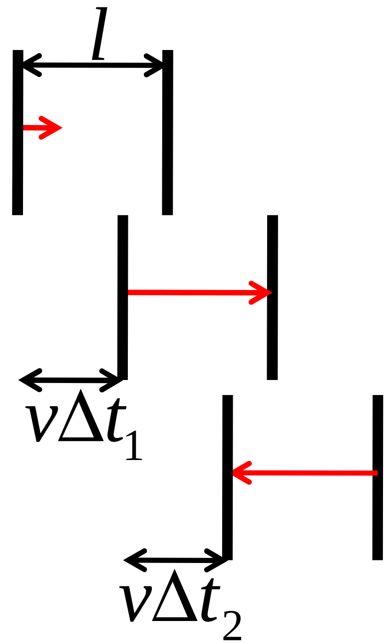


$$2\sqrt{l_0^2 + (v\Delta t / 2)^2} = c\Delta t$$

$$\Rightarrow \quad \Delta t = \gamma\Delta\tau > \Delta\tau \;\;,\;\; \gamma = \frac{1}{\sqrt{1 - v^2 / c^2}} > 1$$

Lorentz length contraction: moving rods contract

- Length of clock in rest frame $l_0 = c\Delta\tau / 2$
- Seen from S:



1. light leaves first mirror
2. reflects at second mirror
3. back at first mirror

$$c\Delta t_1 = l + v\Delta t_1 \quad , \quad c\Delta t_2 = l - v\Delta t_2$$

$$\Delta t = \Delta t_1 + \Delta t_2 = \frac{l}{c-v} + \frac{l}{c+v} = \frac{2l/c}{1-v^2/c^2} =$$

$$= \gamma\Delta\tau = \frac{2l_0/c}{\sqrt{1-v^2/c^2}} \quad \Rightarrow \quad l = l_0\sqrt{1-v^2/c^2} = l_0/\gamma < l_0$$

Lorentz transformation

- Nonrelativistic Galilei transformation

$$x' = x - vt,\ y' = y,\ z' = z,\ t' = t$$

- *x′* seen from S is Lorentz contracted to *x′*/γ

$$x'/\gamma = x - vt \quad \Rightarrow \quad \boxed{x' = \gamma(x - vt)}$$

- No relative speed in *y,z*: $\boxed{y' = y,\ z' = z}$
- Lightspeed=constant gives $x = ct,\ x' = ct'$

$$ct' = \gamma(ct - vx/c) \quad \Rightarrow \quad \boxed{t' = \gamma(t - vx/c^2)}$$

Lorentz invariant interval

- The spacetime interval

$$\Delta s^2 = c^2 t^2 - x^2 - y^2 - z^2$$

is a Lorentz invariant:
it has the same value for all inertial observers

Minkowski diagram

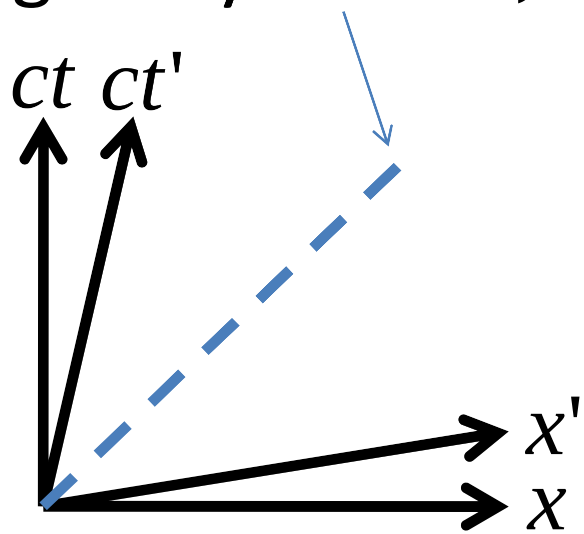
- Line of constant *x'*=0:

$$x' = 0 = \gamma(x - vt) \Rightarrow ct = cx / v$$

- Line of constant *t'*=0:

$$t' = 0 = \gamma(t - vx / c^2) \Rightarrow ct = vx / c$$

- Equation of light ray: $ct = x, ct' = x'$



Doppler shift

- Consider a light source that approaches/leaves an observer in S with speed v.
- Frequency shift

  source approaching: $\nu = \sqrt{\frac{1+v/c}{1-v/c}}\nu' > \nu'$

  source leaving: $\nu = \sqrt{\frac{1-v/c}{1+v/c}}\nu' < \nu'$

- Example: red shift of light from distant galaxies that rapidly move away

Velocity addition

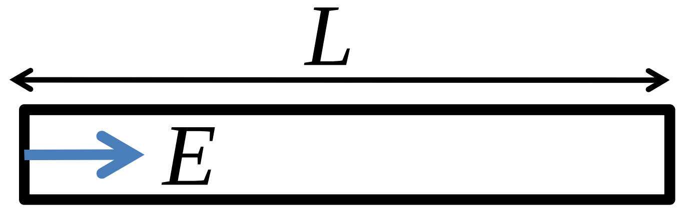
- Consider a particle with velocity v_p in S and v'_p in S'.
- Relativistic velocity addition:

$$v_p = \frac{v'_p + v}{1 + v v'_p / c^2}$$

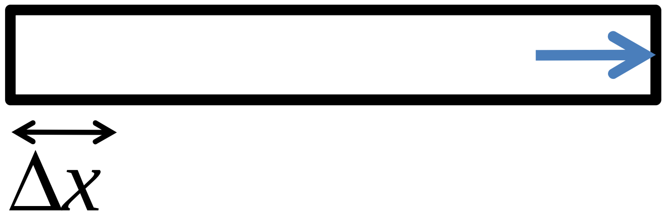
- Example: $v'_p = c \;\Rightarrow\; v_p = \dfrac{c + v}{1 + vc/c^2} = c$

so the speed limit is enforced

E=mc²



- Box (mass *M*) emits light with energy *E=pc* and recoils with *v=-E/Mc*



- Light reaches opposite wall at $\Delta t = L / c$
- Box moves the distance $\Delta x = v\Delta t = -EL / Mc^2$
- But MC cannot move $\Rightarrow$ light transmits mass *m*

$$0 = \Delta x_{MC} = mL + M\Delta x = mL - MEL / Mc^2$$

$$\Rightarrow \quad E = mc^2$$

Relativistic mechanics

- Newtons law $F = \dot{p}$
- Relativistic momentum $p = \gamma m v$
- Total energy $E = \gamma m c^2$
- Energy-momentum relation $E^2 = p^2 c^2 + m^2 c^4$
- Kinetic energy $T = E - mc^2 \approx \frac{1}{2} m v^2 \quad \text{for} \;\; v << c$
- Ultrarelativistic regime $v \approx c \quad \Rightarrow \quad E \approx pc$
- Massless particles have $E = pc$
  and travel at the speed of light
- Lorentz invariant energy-momentum interval

  $$\frac{1}{c^2} E^2 - p^2 = m^2 c^2$$

  Combines energy and momentum conservation
- Maxwells equations already relativistically covariant