

Critical properties of Bose-glass superconductors

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Abstract. – We study vortex lines in high-temperature superconductors with columnar defects produced by heavy-ion irradiation. We reconsider scaling theory for the Bose-glass transition with tilted magnetic fields, and propose, *e.g.*, a new scaling form for the shape of the Bose glass phase boundary, which is relevant for experiments. We also consider Monte Carlo simulations for a vortex model with a screened interaction. Critical exponents are determined from scaling analysis of Monte Carlo data for current-voltage characteristics and other quantities. The dynamic critical exponent is found to be $z = 4.6 \pm 0.3$.

A rich variety of new vortex phase transitions has been established in high-temperature superconductors [1]. Bose-glass physics was originally suggested, by Nelson and Vinokur [2], to apply for vortex phase transitions in systems with artificially introduced columnar defects, produced as permanent damage tracks from heavy-ion irradiation of the sample. Columnar defects act as optimal pinning centers, leading to a considerable increase of critical currents and fields, as compared to the unirradiated sample [3,2,4]. Bose-glass theory makes a set of distinct predictions, *e.g.*, for the dynamic scaling of vortex transport properties, and the response to tilted magnetic fields [2,5,6]. Such behavior has been observed in various experiments [7-12].

Vortex lines in 3D superconductors with columnar defects can be mapped to imaginary time world lines of bosons in 2+1 dimensions on a disordered substrate [2]. The superconducting glass phase for the vortex lines corresponds to the insulating Bose-glass (BG) phase for the bosons. The dissipative vortex line liquid phase corresponds to the superconducting phase for bosons. This mapping gives useful information about the equilibrium properties in the columnar defect problem [13]. However, dynamical properties do not follow from the mapping. A tilted magnetic field H_{\perp} with respect to the columns enters like an imaginary vector potential for the bosons [2], leading to a localization problem in non-Hermitian quantum mechanics, which has received considerable attention recently [14,15]. The vortices want to stay localized on the columns, which results in a transverse Meissner effect with a divergent tilt modulus c_{44} . The BG phase boundary in the (T, H_{\perp}) -plane has a sharp cusp at $H_{\perp} = 0$, which

distinguishes the Bose glass from the isotropic point defect vortex glass which has a smooth phase boundary [2]. Such a feature in the transition line is seen in experiments [8,9,12], and provides support for the Bose-glass theory. Other types of correlated disorder, like splayed columnar defects [16], and planar defects [2], also strongly influence the properties of the glass phase.

In this paper we reconsider scaling theory for the Bose-glass transition for the case of tilted magnetic fields, which leads to certain important modifications of predictions in earlier work [2], *e.g.*, for the form of the BG phase boundary. We further present Monte Carlo (MC) simulations for various thermodynamic and dynamic quantities, which verifies the scaling predictions derived below, and give an estimate of the dynamical critical exponent z .

We first discuss scaling theory. A detailed scaling theory has been developed for the superconducting phase transition by Fisher *et al.* [17], and generalized to anisotropic Bose-glass scaling in refs. [2,5,6]. In the present problem it is crucial to correctly distinguish between the scaling of the \mathbf{B} and \mathbf{H} fields. These relations have sometimes been mixed up in the literature. On approaching the transition the correlation lengths in the directions perpendicular and parallel to the columnar defects are assumed to diverge as $\xi \equiv \xi_{\perp} \sim |T - T_{\text{BG}}|^{-\nu}$ and $\xi_{\parallel} \sim \xi^{\zeta}$, respectively, where ζ is an anisotropy exponent. Due to screening of the interaction and the correlated disorder, the correlation volume is anisotropic with $\zeta = 2$ [13]. In case of unscreened long-range interactions we expect instead $\zeta \approx 1$. The correlation time diverges as $\tau \sim \xi^z$ where z is the dynamical critical exponent. The vector potential enters in the combination $\nabla - (2\pi i/\Phi_0)\mathbf{A}$, and therefore scales as $A_{\perp} \sim \xi^{-1}$, $A_{\parallel} \sim \xi_{\parallel}^{-1}$. From hyperscaling the free-energy density scales as $f \sim \xi^{1-d-\zeta}$, where d is the dimension, and therefore the current density scales as $J_{\perp} = \frac{\partial f}{\partial A_{\perp}} \sim \xi^{2-d-\zeta}$, $J_{\parallel} = \frac{\partial f}{\partial A_{\parallel}} \sim \xi^{1-d}$, and the electric field as $E_{\perp} = -\frac{1}{c} \frac{\partial A_{\perp}}{\partial t} \sim \xi^{-(1+z)}$, $E_{\parallel} = -\frac{1}{c} \frac{\partial A_{\parallel}}{\partial t} \sim \xi^{-(\zeta+z)}$. The scaling of the magnetic field is obtained from $B = -4\pi \frac{\partial f}{\partial H}$ (or from $\nabla \times \mathbf{H} = \frac{4\pi}{c} \mathbf{J}$), which gives $H_{\perp} \sim \xi^{2-d}$, $H_{\parallel} \sim \xi^{3-d-\zeta}$, and the flux density scales as $B_{\perp} = (\nabla \times \mathbf{A})_{\perp} \sim \xi^{-1-\zeta}$, $B_{\parallel} = (\nabla \times \mathbf{A})_{\parallel} \sim \xi^{-2}$. The appropriate scaling combinations involving the magnetic field are therefore $H_{\perp} \xi^{d-2}$ and $H_{\parallel} \xi^{d-3+\zeta}$, which differs from those found, *e.g.*, in ref. [2]. This has several experimentally relevant consequences as we will discuss below. The linear resistivity scales as

$$\rho_{\perp} = \xi^{d+\zeta-3-z} \tilde{\rho}_{\perp}^{\pm}(H_{\perp} \xi^{d-2}), \quad (1)$$

$$\rho_{\parallel} = \xi^{d-\zeta-1-z} \tilde{\rho}_{\parallel}^{\pm}(H_{\perp} \xi^{d-2}), \quad (2)$$

and the I - V characteristic as

$$E_{\perp} = \xi^{-(1+z)} \tilde{E}_{\perp}^{\pm}(J_{\perp} \xi^{d+\zeta-2}, H_{\perp} \xi^{d-2}), \quad (3)$$

$$E_{\parallel} = \xi^{-(\zeta+z)} \tilde{E}_{\parallel}^{\pm}(J_{\parallel} \xi^{d-1}, H_{\perp} \xi^{d-2}), \quad (4)$$

where $\tilde{\rho}$ and \tilde{E} are scaling functions. For the magnetic permeability we obtain

$$\mu_{\perp} = \xi^{d-3-\zeta} \tilde{\mu}_{\perp}(H_{\perp} \xi^{d-2}), \quad (5)$$

$$\mu_{\parallel} = \xi^{d-5+\zeta} \tilde{\mu}_{\parallel}(H_{\perp} \xi^{d-2}). \quad (6)$$

As an independent check of the correctness of this scaling law we can again use the mapping to dirty bosons in (2+1)D. The permeability μ_{\perp} corresponds to the superfluid density ρ_s for the bosons, whose scaling was derived in ref. [13] with the result $\rho_s \sim \xi^{3-d-\zeta}$. This scaling law coincides with eq. (5) for $d = 3$ [18]. Equation (6) corresponds to the compressibility of the bosons $\kappa \sim \xi^{\zeta-d+1}$, again with agreement for $d = 3$. To obtain the scaling of the BG phase

boundary for small tilt, eq. (5) gives $\mu_{\perp} = t^{-\nu(d-3-\zeta)} \tilde{\mu}_{\perp}(H_{\perp} t^{-\nu(d-2)})$, where $t = |T - T_{\text{BG}}|$, and following ref. [2] we assume that the Bose-glass phase persists up to a finite tilt, so $\tilde{\mu}_{\perp}(x)$ has a singularity at a finite value $x = x_c$. At this point the critical tilt field vanishes when $T \rightarrow T_{\text{BG}}$ as

$$T_{\text{BG}} - T \sim |H_{\perp}|^{1/\nu}, \quad (7)$$

for $d = 3$. Equations (5)-(7) differ from the corresponding ones in ref. [2], and in particular our eq. (7) does not include a factor 3 in the exponent. The shape of the phase boundary still has a cusp at $H_{\perp} = 0$, but not as sharp as the earlier predictions suggested. Another consequence of experimental interest is the scaling relation $\rho_{\perp} \sim H_{\perp}^{z-\zeta}$ for the resistivity at $T = T_{\text{BG}}$.

We now turn to our Monte Carlo simulations. We concentrate on low magnetic fields, $B \lesssim \Phi_0/\lambda^2$, where the vortex interaction is effectively short-ranged, and also restrict our study to fields below the matching field, where the density of columnar defects is greater than the vortex density. Vortex dynamics has been studied previously in simulations of the non-linear current-voltage (I - V) characteristics [5] for finite applied supercurrents. In this paper we present extensive simulation results for the linear resistance, and compare with dynamical scaling of the I - V characteristics. We choose to study the dirty Boson action with an onsite repulsion [13], which gives a coarse-grained representation of the physical system,

$$H = \sum_{\mathbf{r}} \left\{ \frac{1}{2} \mathbf{m}_{\mathbf{r}}^2 - g(\mathbf{r}_{\perp}) m_{\mathbf{r}z} \right\}, \quad (8)$$

where the integer variable $\mathbf{m} = (m_x, m_y, m_z)$ is the vorticity vector on the links of a simple cubic lattice of size $\Omega = L \times L \times L_{\parallel}$ with lattice constant $a = 1$. The partition function is $Z = \text{Tr} e^{-\beta H}$, where the trace is the sum over all vortex line configurations with no open ends, thus satisfying the constraint $\nabla \cdot \mathbf{m} = 0$, and $\beta = 1/T$ is the inverse temperature. Periodic boundary conditions are used in all directions to eliminate surface effects. The columnar defects are aligned in the z -direction, and are modeled as a uniformly distributed random site energy $g(\mathbf{r}_{\perp})$, which is constant in the z -direction. The applied magnetic field is included as a finite fixed net density of vortex lines. We also consider tilted magnetic fields by allowing global vortex line fluctuations in the direction perpendicular to the columns, with a bias term $-\sum_{\mathbf{r}} H_x m_{\mathbf{r}x}$ added to the Hamiltonian. Although not realistic in detail, this model should fall into the universality class of strongly screened vortices with columnar defects and therefore give the critical exponents correctly for low enough magnetic fields.

Our Monte Carlo method for vortex line models has been described elsewhere and is only outlined here [19]. The starting configuration is taken as an assembly of straight vortex lines penetrating the system in the z -direction. The MC moves are attempts to add closed vortex loops around randomly selected elementary plaquettes of the lattice. The trial moves are accepted with probability $1/(1 + e^{\beta \Delta H})$, where ΔH is the total change in energy.

I - V characteristics can be calculated by the following method [5, 19]. Each time a loop is formed it generates a voltage pulse $\Delta Q = \pm 1$ perpendicular to its plane, the sign depending on the orientation of the loop. This leads to a net electric field $E(t) = \frac{\hbar}{2e} J^V(t)$, (in the following we set $\hbar/(2e) = 1$) where the vortex current density at MC time t is given by $J^V(t) = \frac{\Delta Q}{\Delta t}$, and $\Delta t = 1$ for one full sweep through the system, where, on average, an attempt is made to create or destroy one vortex loop on every plaquette of the lattice. The non-linear I - V characteristic can be modeled as the electric field E , due to vortex current response in the presence of a uniform Lorentz force on the vortex lines, proportional to the applied current density J . The linear resistance can be calculated from the equilibrium voltage fluctuations via the Kubo formula [20], $R = \frac{1}{2T} \sum_{t=-\infty}^{\infty} \Delta t \langle V(t)V(0) \rangle$. Here $\langle \dots \rangle$ denotes thermal average and $[\dots]$ disorder average.

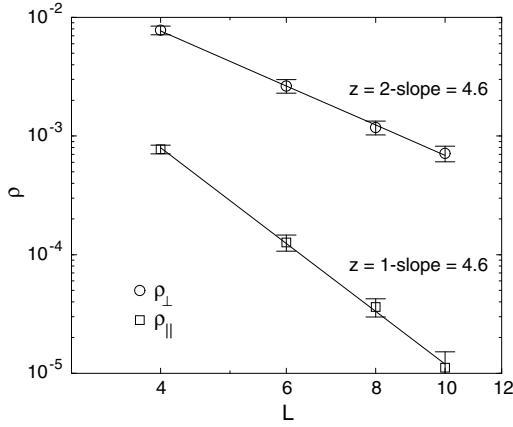


Fig. 1

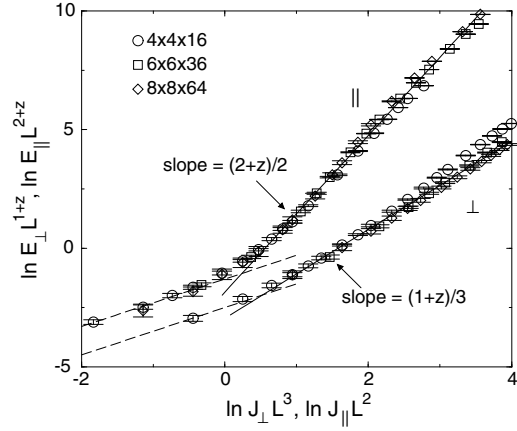


Fig. 2

Fig. 1. – MC results for the linear resistivity *vs.* system size L at the Bose-glass transition in the directions parallel and perpendicular to the columnar defects. From the slope of the curves the dynamical critical exponent is estimated to be $z \approx 4.6$ for both directions.

Fig. 2. – Non-linear current-voltage characteristics at the Bose-glass critical point. Dashed lines have slope one (Ohmic response). Solid lines have slope corresponding to the value $z = 4.6$ from the linear resistivity in fig. 1.

To study the response to tilt, we consider an ensemble with fluctuating winding number. We achieve this by including global MC moves where vortex lines are inserted across the whole system in the direction perpendicular to the columns. The uniform part of the flux density is proportional to the winding number, $B_\nu = \frac{W_\nu}{\Omega} = \frac{1}{\Omega} \sum_{\mathbf{r}} m_{\mathbf{r}\nu}$. In a tilted magnetic field $[\langle B_\perp \rangle]$ is non-zero above the BG temperature and zero below. The magnetic permeability in the direction perpendicular to the columns is given by

$$\mu_\nu = \frac{\partial [\langle B_\nu \rangle]}{\partial H_\nu} = \frac{\Omega}{T} \left[\langle B_\nu^2 \rangle - \langle B_\nu \rangle^2 \right], \quad (9)$$

and is related to the tilt modulus by $c_{44} \sim 1/\mu_\perp$. In the analogy to dirty bosons, μ_\perp corresponds to the renormalized superfluid density ρ_s of the bosons.

The details of our MC calculations are as follows. We use a net density of vortex lines corresponding to filling of $f = 1/2$ flux quanta per plaquette. We use a disorder potential g uniformly distributed in $[0, 1]$. The model, eq. (8), has been extensively studied in the context of dirty bosons, and has a glass transition at $T_{BG} \approx 0.248$ [13] (in case of no tilted magnetic field). In finite systems the scaling functions obtain extra arguments ξ/L and $\xi_\parallel/L_\parallel$. To enable finite-size scaling of MC data, we use system sizes $L_z = cL^\zeta$, making the ratio L_\parallel/L^ζ a constant, which eliminates one argument from the scaling functions. We use $\zeta = 2$, $c = 1$, and lattice sizes $L = 4, \dots, 16$. For each realization of the columnar disorder, up to $2 \cdot 10^4$ MC sweeps were discarded for warm-up, and measurements taken during up to $2 \cdot 10^6$ sweeps. Up to 10^3 disorders were used to obtain small statistical errors in the disorder averages.

We now turn to our MC results, starting with the magnetic field applied along the columns. Figure 1 shows a log-log plot of the linear resistivities ρ_\perp in the direction perpendicular to the columnar defects, and ρ_\parallel parallel to the columns, *vs.* system size L at $T = T_{BG}$. Power law

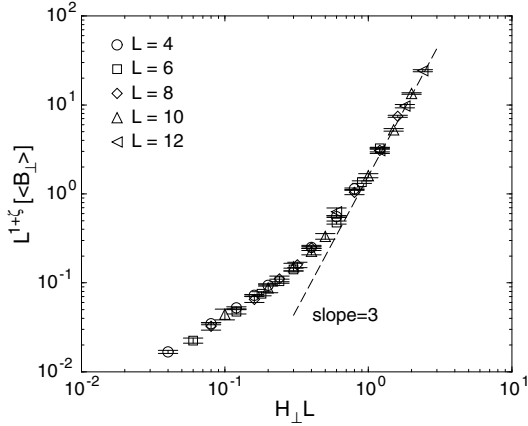


Fig. 3

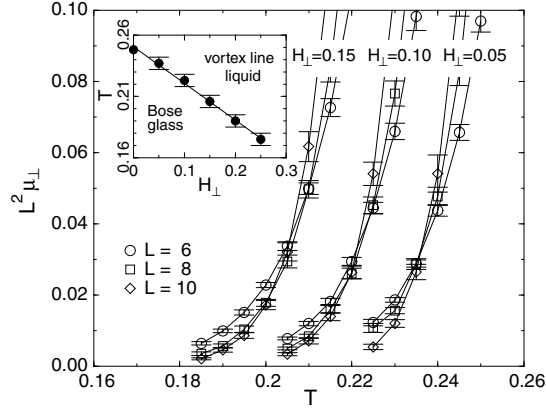


Fig. 4

Fig. 3. – Finite-size scaling of MC data for B_{\perp} for various sizes L and applied perpendicular fields H_{\perp} at $T = T_{\text{BG}}$. The dashed line indicates the form $B_{\perp} \sim H_{\perp}^{1+\zeta}$, with $\zeta = 2$, for $H_{\perp} L > 1$.

Fig. 4. – MC data for magnetic permeability μ_{\perp} in a tilted magnetic field. The critical temperature $T_c(H_{\perp})$ is roughly estimated as the point where the curves for different L intersect (filled circles in the inset). Inset: The resulting BG phase boundary. The solid straight line corresponds to a power law $T_{\text{BG}} - T \sim |H_{\perp}|^{1/\nu}$, with $\nu = 1.0$.

fits to the data (solid lines) allow z to be determined according to eqs. (1), (2), which gives $z = 4.6 \pm 0.3$ in both directions.

Figure 2 shows a finite-size scaling data collapse according to eqs. (3), (4) of the non-linear current-voltage characteristics for currents applied perpendicular and parallel to the columnar defects, at $T = T_{\text{BG}}$. In the limit of small currents we observe Ohmic response when the non-linear response length scale exceeds the system size. In a finite range of sufficiently small currents the I - V characteristics show power law behavior as indicated by the straight lines in the plot. The solid straight lines are given by the scaling forms in eqs. (3) and (4), using the value $z = 4.6$ determined in fig. 1 from the linear resistivity. For higher currents there are clearly visible deviations, and a crossover to different power laws takes place. If this range of large currents is fitted to eqs. (3), (4) one has to assume different values of z in the \perp and \parallel directions: $z_{\perp} \approx 6$, $z_{\parallel} \approx 4$, in agreement with ref. [5].

We next consider tilted magnetic fields. As before a constant field was applied parallel to the columns, corresponding to a half filling, and in addition we apply a small perpendicular field H_{\perp} . We first consider the effect of H_{\perp} on B_{\perp} at the Bose glass temperature $T = T_{\text{BG}}$, shown in fig. 3. The data collapse for different system sizes and tilts verifies the scaling form $B_{\perp} = L^{-1-\zeta} \tilde{B}_{\perp}(H_{\perp} L^{d-2})$. The dashed line corresponds to the power law scaling form $B_{\perp} \sim H_{\perp}^{1+\zeta}$ for scaling variable $H_{\perp} L \gtrsim 1$. To locate the BG phase boundary as the magnetic field is tilted, we use eq. (5), which, for finite systems and $d = 3$, gives $\mu_{\perp} = L^{-\zeta} \tilde{\mu}_{\perp}([T - T_c(H_{\perp})] L^{1/\nu''})$. Here ν'' is the correlation length exponent for finite tilt fields, which belongs to a separate universality class [2]. At the critical temperature in a finite system, this should scale as $\mu_{\perp} \sim L^{-\zeta}$ for large enough L . In fig. 4 we use the crossing point for MC data for small system sizes as a crude estimate of the critical temperature. However, rather large corrections to scaling are expected due to the closeness of the BG fixed point, and we were therefore not able to determine ν'' . The inset in fig. 4 shows the transition

temperatures (filled circles) *vs.* H_{\perp} . The solid straight line represents eq. (7), using the value $\nu = 1.0$ for the BG transition [13]. Thus the form of the BG phase boundary given by eq. (7) seems consistent with the simulation.

Finally we will compare our results with experiments. The transport measurements for $\text{Tl}_2\text{Ba}_2\text{CaCu}_2\text{O}_8$ (Tl-2212) thin films with columnar defects and zero tilt [11] obtained $z \approx 4.9$, $\nu \approx 1.1$, $\zeta \approx 1.9$, which closely agree with our exponents. Experiments for Tl-2212 with tilted magnetic fields [7] obtained $z'' \approx 4.4$, $\nu'' \approx 1.8$, where the double prime indicates exponents for the tilted fixed point. A heavily twinned YBCO single crystal, without artificial columnar defects, gave [12] $\nu(z-2) \approx 2.8$, for zero tilt, which is close to our number $\nu(z-2) \approx 2.6$. Quite different experimental results were found in ref. [8]. Experiments on $\text{YBa}_2\text{Cu}_3\text{O}_7$ (YBCO) single crystals with columnar defects in low magnetic fields (0-6.3 kOe) [8] suggest $z \approx 2.2$, $\nu \approx 1$, $\zeta \approx 1$. The isotropic scaling indicates that the vortex interaction is not effectively screened [13], and a model with a long-range vortex interaction may be suitable. We have also done some (limited) simulations for a model with unscreened long-range interactions, and find a dynamical exponent $z \approx 2$, which is considerably smaller than the one for strong screening. Experiments with tilted magnetic fields [12,9,8] show a sharp cusp in the BG phase boundary, in agreement with the Bose-glass theory. We note, however, that the actual shape of the BG phase boundary in the figures in refs. [12,9,8], appears to fit reasonably well our $T_{\text{BG}}(0) - T_{\text{BG}}(H_{\perp}) \sim |H_{\perp}|^{1/\nu}$ for small H_{\perp} , with exponent $1/\nu \approx 1.0$ instead of $1/3\nu$ from ref. [2]. More experimental data for the precise shape of the BG phase boundary and for other quantities would be useful to test our new scaling relations.

In summary, we have analyzed the Bose-glass transition in superconductors with columnar defects by means of scaling theory and Monte Carlo simulations. For magnetic fields tilted away from the direction of the columns we suggest a form of the BG phase boundary for small tilts, which appears consistent with our simulations and with experiments. Our value $z = 4.6 \pm 0.3$ for the dynamical critical exponent at the BG transition in a strongly screened model is in good agreement with recent experiments.

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